



Closed-Form Outage Expression of Fluid Antenna Systems for Ultra-Reliable Communications

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Abstract: Due to its ability to significantly enhance communication reliability, the fluid antenna system (FAS) has emerged as a promising technology for ultra-reliable communications (URC). However, existing expressions to evaluate the outage probability of FAS are either computationally complex or simplified at the cost of accuracy. To address this issue, we propose an accuracy-controllable approximation method under Rayleigh fading channels. Specifically, we first approximate the fully correlated Rayleigh fading model as a block-correlation Rayleigh model, where the channel gains of ports in different blocks are assumed to be mutually independent, while the channel gains of ports in the same block are considered equally correlated, to simplify the analysis while maintaining accuracy. Then, we propose a novel method to approximate the Bessel function I_0 , introduced by the correlated Rayleigh fading, using an M -term series expansion, where the parameter M provides a tunable trade-off between approximation accuracy and computational complexity. Based on this approximation, a closed-form expression for the outage probability of FAS is derived. Simulation results demonstrate that our derived expression offers low computational complexity for evaluating the FAS outage probability while maintaining excellent approximation accuracy in the medium-to-high signal-to-noise ratio (SNR) range.

Keywords: ultra-reliable communication; fluid antenna system; approximation; outage probability

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1 Introduction

Ultra-reliable communications (URC) is a key Beyond 5G (B5G) innovation that supports mission-critical applications, e. g. , autonomous vehicles and remote surgery, by ensuring over 99. 999% transmission reliability^[1-2]. Such ultra-reliability is often achieved through diversity techniques, particularly by realizing spatial diversity with multiple antennas in Multiple-Input Multiple-Output (MIMO) technology to combat signal fading^[3]. However, MIMO requires the minimum antenna spacing of half the wavelength to prevent electromagnetic coupling^[4]. This physical constraint makes it difficult to implement in highly miniaturized devices. To address this challenge, the fluid antenna system (FAS) has emerged. The core of FAS lies in its capability for dynamic reconfiguration. Specifically, this refers to a system

that can automatically switch the antenna position among N predefined locations, known as ports, within a given space^[5]. By selecting the port with the best channel condition, the system effectively leverages spatial diversity to maximize the received signal strength.

Accurate yet tractable performance analysis for FAS over Rayleigh fading is an important research direction, but it remains challenging due to the inherent spatial correlation among the fading channels. According to the classical Jakes' model, the spatial correlation between any two ports depends on the distance between the two ports Δd and the wavelength λ , and is characterized by the zero-order Bessel function of the first kind, i.e., $J_0(2\pi\Delta d/\lambda)$. Until now, considerable research efforts have been devoted to addressing this issue.

The authors in Ref. [6] proposed a reference point model where the first port is selected as the reference port and the spatial correlation between the k -th port and the reference port is characterized by Jakes' model. Based on this model, the authors derived the ergodic capacity and a corresponding lower

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bound. Subsequently, building directly upon the same reference point framework, the work in Ref. [7] derived the exact joint distribution of the signal envelopes for all ports, which enabled the derivation of the outage probability. Considering the outdated channel state information (CSI), the authors in Ref. [8] also employed the reference point model to develop an analytical framework for the outage probability of FAS. Although the reference point model has been widely used to derive expressions of the performance metric, it only captures the correlation of each port with a single reference and ignores correlations among other ports, leading to an overestimation of the achievable transmission performance. To address this limitation, Khammassi et al. introduced a fully correlated model^[9]. By performing an eigenvalue decomposition on the channel covariance matrix prescribed by Jakes' model, the fully correlated model in Ref. [9] expresses each channel coefficient as a weighted sum of the eigenvectors, where the weights are independent Gaussian random variables scaled by the square roots of the corresponding eigenvalues. This construction rigorously ensures that the correlation between any two ports is consistent with Jakes' model, thereby establishing a more accurate theoretical foundation for FAS performance analysis. Based on this model, the authors derived an approximate expression for the outage probability in single-integral form by retaining only the dominant eigenvalues of the channel covariance matrix to simplify the analysis^[9]. Although the fully correlated model addresses the limitations of the reference point model, it imposes high computational complexity. To address this issue, the authors in Ref. [10] proposed a spatial block-correlation model. The key idea of this model is the "structured approximation" using a block-diagonal matrix to capture the dominant spatial correlations. This construction is guided by the finding that the number of significant eigenvalues of the true covariance matrix is approximately $2W$ for a 1D fluid antenna of normalized length W . By grouping the ports into independent blocks, each designed to approximate one dominant eigenvalue, the model effectively decouples the system. This results in mutual independence between blocks and equal correlation within a block, thereby significantly simplifying subsequent mathematical analysis and computation. A persistent bottleneck across these models, however, is the lack of a closed-form expression for the outage probability. This is because the Rayleigh fading model leads to a probability density function (PDF) that involves an exponential term. When evaluating the outage probability for selection combining, this exponential term becomes coupled with the modified Bessel function I_0 , forming the integral kernel of the Marcum Q-function. It is the presence of this particular function within the integrals that prevents a closed-form evaluation. The lack of a closed-form solution not only hinders rapid performance evaluation but also becomes a bottleneck when extending FAS to more complex and promising frontiers. For instance, in multi-user systems, efficient power allocation and interference management rely on trac-

table performance metrics^[11]. Furthermore, the integration of FAS with advanced architectures, such as reconfigurable holographic surfaces, demands efficient analytical tools for joint optimization^[12]. Thus, the pursuit of a closed-form outage probability expression is motivated not only by theoretical elegance but also by practical necessity in advancing these cutting-edge applications.

To obtain a closed-form outage probability expression, we employ the same system model as in Ref. [10]. Within this framework, our main contributions are as follows:

- A novel approximation is introduced by applying an M -term Taylor series expansion to the zero-order modified Bessel function of the first kind, I_0 , which constitutes the core of the Marcum Q-function's integral kernel. The proposed closed-form approximation achieves high computational efficiency by eliminating numerical integration, enabling rapid parameter sweeps and real-time analysis. Furthermore, it provides tunable accuracy through adjustment of the M -term Taylor series, allowing for a flexible balance between precision and complexity.
- By integrating this approximation with the multinomial theorem, we derive an approximate closed-form expression for the outage probability, establishing an efficient and accurate tool for FAS performance evaluation.
- The theoretical findings are rigorously validated through Monte Carlo simulations, confirming the validity and precision of our analytical approach.

2 System Model

We consider the same point-to-point system model as in Ref. [10], with a single-user FAS receiver, as illustrated in Fig. 1.

The array consists of N uniformly spaced ports arranged linearly on a dielectric holder of length $W\lambda$, where λ denotes the wavelength of the transmitted signals.

We assume that all communication links experience Rayleigh fading. Consequently, the signal received at the k -th port of the fluid antenna can be expressed as

$$y_k = g_k x + \eta_k \quad (1),$$

where x is the transmitted data symbol with energy P , g_k is the

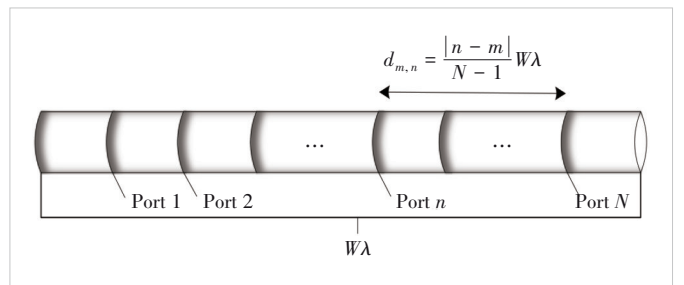


Figure 1. Geometry of the linear FAS with N candidate ports

channel coefficient at the k -th port with $E[|g_k|^2] = \sigma^2$, and $\eta_k \sim \mathcal{CN}(0, \sigma_\eta^2)$ is the additive white Gaussian noise (AWGN). Next, we define the average signal-to-noise ratio (SNR) as $\Gamma \triangleq \sigma^2 \frac{E[|x|^2]}{\sigma_\eta^2}$. For simplicity, and without loss of generality, we set $\sigma^2 = 1$ in the following derivations.

The spatial correlation among the FAS ports is characterized using the block-diagonal model of Ref. [10]¹, whose specific construction is detailed below.

1) Spatial coherence region partitioning

The total N FAS ports are partitioned into B independent spatial blocks, with each block containing $L(i)$ ports (satisfying $\sum_{i=1}^B L(i) = N$). The inter-port spacing within each block is smaller than the spatial coherence distance, while the spacing between blocks exceeds the coherence distance.

2) Intra-block correlation matrix

The covariance matrix of the i -th block follows a constant correlation structure:

$$\mathbf{A}_b = \begin{pmatrix} 1 & \mu_i^2 & \cdots & \mu_i^2 \\ \mu_i^2 & 1 & \cdots & \mu_i^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mu_i^2 & \mu_i^2 & \mu_i^2 & 1 \end{pmatrix} \quad (2),$$

where $\mu_i^2 \rightarrow \Gamma$.

3) Inter-block independence

The global covariance matrix exhibits a block-diagonal structure, and this configuration models the FAS channel as B independent spatial degrees of freedom, where B is determined by the antenna aperture size W ($B \approx 2W$ for linear FAS)^[10].

3 Outage Performance Analysis

In this section, we derive a closed-form expression for the outage probability of FAS with optimal port selection. The system adopts selection combining (SC) that dynamically selects the antenna port exhibiting maximum instantaneous received signal power:

$$h_{\text{FAS}} = \max\{h_1, \dots, h_N\} \quad (3),$$

where $h_n = |g_n|^2$. Thus, we can write $\{h_n\}$ as

$$h_n = \left(\sqrt{1 - \mu_n^2} x_n + \mu_n x_0 \right)^2 + \left(\sqrt{1 - \mu_n^2} y_n + \mu_n y_0 \right)^2 \quad (4).$$

For the single-user scenario, the resulting outage probability is given by Eq. (43) in Ref. [10] as:

$$P_{\text{out}}(\gamma) = P\left(\max_n \{(1 - \mu^2)X_n\} < \gamma\right) \quad (5),$$

where X_n is defined as

$$X_n = \frac{h_n}{1 - \mu_n^2} = \left(x_n + \frac{\mu_n}{\sqrt{1 - \mu_n^2}} x_0 \right)^2 + \left(y_n + \frac{\mu_n}{\sqrt{1 - \mu_n^2}} y_0 \right)^2 \quad (6).$$

Based on the block-correlation framework established in Ref. [10], the outage probability of the considered FAS is given by Lemma 2 in Ref. [10] as:

$$P_{\text{out}}(\gamma) = \prod_{b=1}^B \int_0^\infty \frac{1}{2} e^{-\frac{r_0}{2}} \times \prod_{k=1}^{L(b)} \left[1 - Q_1 \left(\sqrt{\frac{\mu^2 r_0}{1 - \mu^2}}, \sqrt{\frac{\gamma}{1 - \mu^2}} \right) \right] dr_0 \quad (7).$$

However, the expression in Eq. (7) remains intractable for deriving a closed-form solution. The key obstacle is the presence of the Marcum Q-function $Q_1 \left(\sqrt{\frac{\mu^2 r_0}{1 - \mu^2}}, \sqrt{\frac{\gamma}{1 - \mu^2}} \right)$

within the integral, which itself is defined by an integral containing a Bessel function. To overcome this fundamental limitation, we propose a novel approximation method that yields a closed-form expression for the outage probability.

The primary challenge in obtaining a closed-form solution stems from the lack of standard integration techniques for the composite structure of the Marcum Q-function. Therefore, we expand it as follows:

$$P_{\text{out}}(\gamma) = \prod_{b=1}^B \int_0^\infty \frac{1}{2} e^{-\frac{r_0}{2}} \prod_{k=1}^{L(b)} \int_0^{\frac{\gamma}{1 - \mu^2}} \frac{1}{2} e^{-\frac{t_k + \frac{\mu^2}{1 - \mu^2} r_0}{2}} \times I_0 \left(\sqrt{\frac{\mu^2 r_0 t_k}{1 - \mu^2}} \right) dt_k dr_0 \quad (8).$$

The challenge of obtaining a closed-form expression persists with Eq. (8), again due to the non-elementary Bessel function I_0 and the nested integral. To overcome this, we apply a Taylor series expansion to $I_0(z)$. The general M -term Taylor series is given by:

$$I_0(z) = \sum_{m=0}^{M-1} \frac{z^{2m}}{2^{2m} (m!)^2} \quad (9).$$

¹ The block-diagonal model is chosen as it strikes a balance between accuracy and analytical tractability. While this model offers a representative structure for derivation, the proposed analytical framework, which employs Taylor series expansion and multinomial theorem to handle the outage probability integral, constitutes the primary contribution and is readily generalizable to other spatial correlation models.

By substituting this expansion into Eq. (8), we obtain a new expression that is amenable to analytical integration:

$$P_{\text{out}}(\gamma) \approx \prod_{b=1}^B \int_0^\infty \frac{1}{2} e^{-\frac{r_0}{2}} \prod_{k=1}^{L(b)} \int_0^{\frac{\gamma}{1-\mu^2}} \frac{1}{2} e^{-\frac{t_k + \frac{\mu^2}{1-\mu^2} r_0}{2}} \times \left[\sum_{m=0}^{M-1} \frac{1}{4^m (m!)^2} \left(\frac{\mu^2 r_0 t_k}{1-\mu^2} \right)^m \right] dt_k dr_0 \quad (10).$$

To illustrate the core idea with clarity and to derive a compact, computationally efficient approximation, we first present the specific case of retaining only the first two terms (i.e., $M = 2$) of the series:

$$P_{\text{out}}(\gamma) \approx \prod_{b=1}^B \int_0^\infty \frac{1}{2} e^{-\frac{r_0}{2}} \prod_{k=1}^{L(b)} \int_0^{\frac{\gamma}{1-\mu^2}} \frac{1}{2} e^{-\frac{t_k + \frac{\mu^2}{1-\mu^2} r_0}{2}} \times \left(1 + \frac{\mu^2 r_0 t_k}{4(1-\mu^2)} \right) dt_k dr_0 = \prod_{b=1}^B \int_0^\infty \frac{1}{2^{1+L(b)}} e^{-\left(\frac{r_0}{2} + \left(\frac{\mu^2 r_0}{2(1-\mu^2)} \right)^{L(b)} \right)} \prod_{k=1}^{L(b)} \left[\int_0^{\frac{\gamma}{1-\mu^2}} e^{-\frac{t_k}{2}} dt_k + \int_0^{\frac{\gamma}{1-\mu^2}} e^{-\frac{t_k}{2}} \frac{\mu^2 r_0 t_k}{4(1-\mu^2)} dt_k \right] dr_0 \quad (11).$$

Then we evaluate the inner integral:

$$\int_0^{\frac{\gamma}{1-\mu^2}} e^{-\frac{t_k}{2}} dt_k = 2 \left(1 - e^{-\frac{\gamma}{2(1-\mu^2)}} \right) \int_0^{\frac{\gamma}{1-\mu^2}} e^{-\frac{t_k}{2}} \frac{\mu^2 r_0 t_k}{4(1-\mu^2)} dt_k = \frac{\mu^2 r_0}{(1-\mu^2)} \left(1 - \left(1 + \frac{\gamma}{2} \right) e^{-\frac{\gamma}{2(1-\mu^2)}} \right) \quad (12).$$

Substituting Eq. (12) into the outer integral:

$$P_{\text{out}}(\gamma) \approx \prod_{b=1}^B \frac{1}{2^{1+L(b)}} \int_0^\infty e^{-\left(\frac{r_0}{2} + \frac{\mu^2 L(b)}{2(1-\mu^2)} \right)} \times \left[\prod_{k=1}^{L(b)} \left(2 \left(1 - e^{-\frac{\gamma}{2(1-\mu^2)}} \right) + \frac{\mu^2 r_0}{(1-\mu^2)} \left(1 - \left(1 + \frac{\gamma}{2} \right) e^{-\frac{\gamma}{2(1-\mu^2)}} \right) \right) \right] dr_0 \quad (13).$$

For computational simplicity, we define $\alpha = \frac{1}{2} + \frac{\mu^2 L(i)}{2(1-\mu^2)}$, $D = \frac{\gamma}{2(1-\mu^2)}$, $E = 2(1 - e^{-D})$, $F = \frac{\mu^2}{(1-\mu^2)}(1 - (1 + D)e^{-D})$, so that

$$P_{\text{out}}(\gamma) = \prod_{b=1}^B \frac{1}{2^{1+L(b)}} \int_0^\infty e^{-\alpha r_b} [E + F r_b]^{L(b)} dr_b \quad (14).$$

Next, the complex integral is transformed into a manageable series through binomial expansion. After interchanging the summation and integration, the integral values are computed using the Gamma function. Based on the independence assumption, the final expression is obtained in a product form:

$$P_{\text{out}}(\gamma) \approx \prod_{b=1}^B \frac{1}{2^{1+L(b)}} \sum_{k=0}^{L(b)} \binom{L(b)}{k} E^{L(b)-k} F^k \frac{\Gamma(k+1)}{\alpha^{k+1}} \quad (15).$$

Having established the closed-form approximation for $M = 1$, we now extend the analysis to an arbitrary number of terms M . The derivation for the general case, starting from Eq. (10), is considerably more involved, as it requires additional analytical techniques, such as the application of the multinomial theorem, to handle the generalized series expansion. The resulting general closed-form expression for the outage probability is given by:

$$P_{\text{out}}(\gamma) = \prod_{b=1}^B \frac{1}{2^{L(b)+1}} \sum_{k_0+k_1+\dots+k_M=L(b)} \binom{L(b)}{k_0, k_1, \dots, k_M} \times \prod_{m=0}^{M-1} \frac{d_m^{k_m} \Gamma(s+1)}{\alpha^{s+1}} \quad (16),$$

where d_m represents the contribution weight corresponding to different expansion terms in the outage probability expression; s is a summation exponent, which computes the weighted sum of the orders of all Taylor expansion terms under a specific exponent combination $\{k_0, k_1, \dots, k_M\}$. The parameters in Eq. (16) are defined as follows:

$$d_m = \frac{2^{-m+1}}{(m!)^2} \left(\frac{\mu^2}{1-\mu^2} \right)^m \gamma(j+1, D), \quad s = \sum_{j=0}^{M-1} j k_j \quad (17).$$

The detailed proof of the above result is provided in Appendix A.

To quantitatively validate the low complexity of the proposed closed-form expression, we compare its computational efficiency with Eq. (7) via both theoretical analysis and MATLAB simulations. First, the computational complexity of Eq. (7) scales as $O\left(X_1 \cdot X_2 \cdot \sum_{b=1}^B L(b)\right)$, where X_1 and X_2 denote the numbers of discretization points for the outer and inner integrals (typically $X_1, X_2 \geq 10^3$ to ensure accuracy). In contrast, the complexity of Eq. (16) is given by $O\left(\binom{L(i)+M}{M} \cdot M\right)$. The binomial coefficient grows only polynomially with $L(i)$, leading to a substantially lower complexity compared with the numerical benchmark. Fig. 2 presents a runtime comparison between the proposed method (for N in

$\{5, 10\}$ and M in $\{2, 4, 6\}$ and the numerical benchmark in Eq. (7), conducted in MATLAB.

A direct comparison reveals that, for the same port count N , the runtime of the proposed closed-form method (Figs. 2a – 2c and 2e – 2g) is consistently and significantly lower than that of the numerical integration method (Figs. 2d and 2h), irrespective of the Taylor expansion term count M , strongly corroborating the theoretical complexity analysis.

To provide rigorous theoretical guarantees for the proposed approximation method, we present here the error bound analysis and convergence proof.

1) Error bound analysis

First, we define $T_M(x)$ as the Taylor expansion truncated after the M -th term:

$$T_M(x) = \sum_{m=0}^{M-1} \frac{x^{2m}}{2^{2m}(m!)^2} \quad (18).$$

The truncation error is defined as

$$\varepsilon_1(x, M) = I_0(x) - T_M(x) \quad (19).$$

Theorem 1 (Truncation Error Bound): For any $x \geq 0$ and positive integer M ,

$$\varepsilon_1(x, M) \leq \frac{x^{2M}}{2^{2M}(M!)^2} I_0(x) \quad (20).$$

2) Propagation to outage probability error

Let $P_{\text{out}}^{\text{exact}}$ denote the true outage probability under the block-correlation model, and $P_{\text{out}}^{(M)}$ denote the approximation using M Taylor terms.

Theorem 2 (Outage Probability Error Bound): Let $\varepsilon_{p_{\text{out}}}$ denote the outage probability error, which is defined as

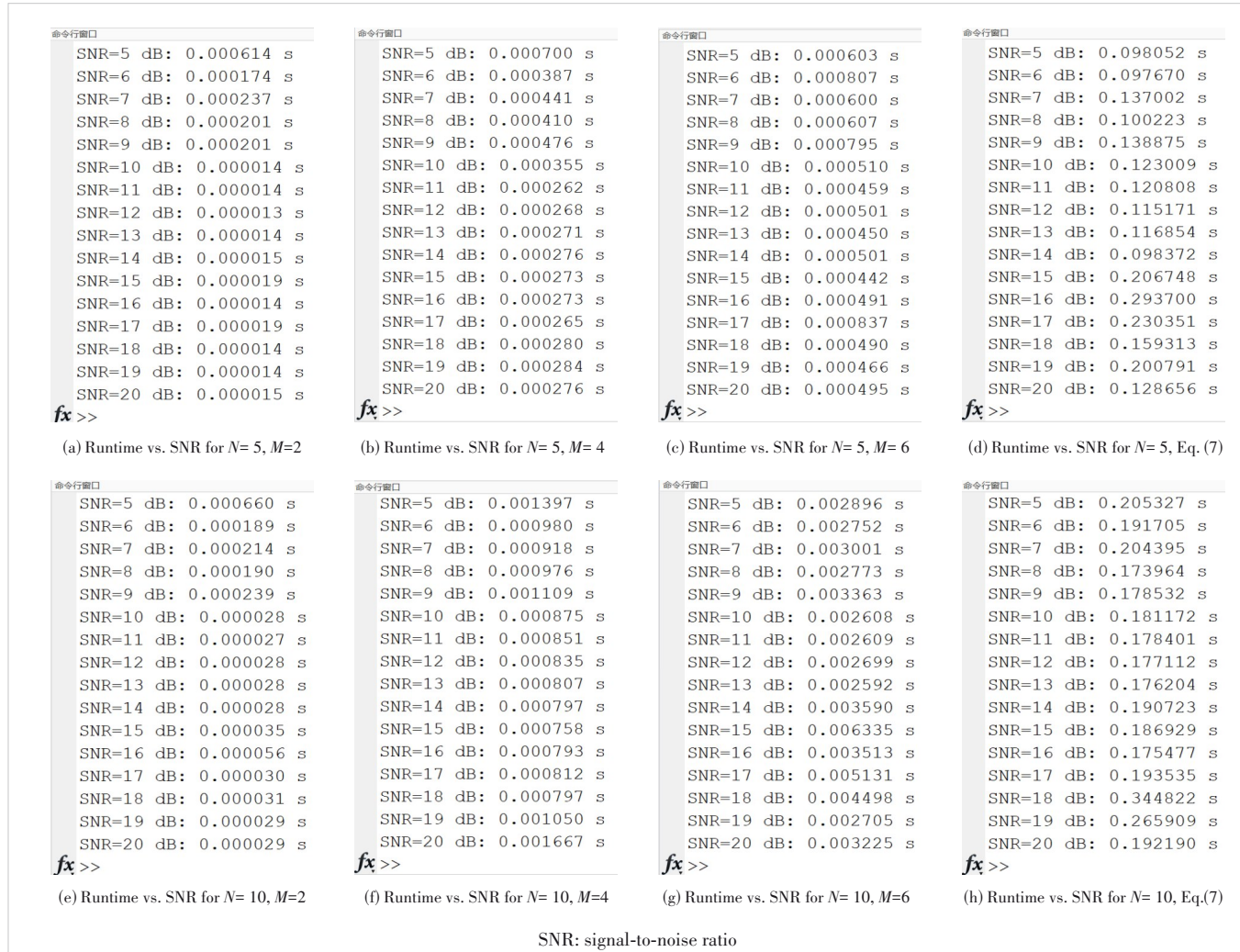


Figure 2. Runtime comparison for different numbers of ports and Taylor expansion terms versus SNR

$$\varepsilon_{P_{\text{out}}} = \left| P_{\text{out}}^{\text{exact}} - P_{\text{out}}^{(M)} \right| \leq \frac{\Gamma^M}{(M!)^2} \cdot C(M, \Gamma) \quad (21),$$

where

$$C(M, \Gamma) = \int_0^\infty e^{-2t} t^M I_0(2\sqrt{\Gamma t}) dt \quad (22),$$

is a finite constant for given system parameters.

3) Convergence proof

Theorem 3 (Convergence): As $M \rightarrow \infty$, the approximate outage probability converges to the true value:

$$\lim_{M \rightarrow \infty} P_{\text{out}}^{(M)} = P_{\text{out}}^{\text{exact}} \quad (23).$$

The detailed proof of the above theorem is provided in Appendix B.

4 Simulations

We now validate our theoretical analysis with computer simulations. For the sake of presentation, we consider the following parameters: $\mu^2 = 0.95$, $W = 1$, $\gamma = 2$, $N \in \{5, 10\}$.

Fig. 3 depicts the relative approximation error, which is defined as $\varepsilon_{\text{rel}} = (\text{simulation result} - \text{analytical result}) / \text{simulation result}$ [13], between the logarithmic forms of the series expansion of I_0 and its original expression given in Eq. (7). This error is plotted against the number of terms M included in the expansion, for different SNR values.

The simulation results validate both the convergence and efficiency of the proposed method across different operating conditions. First, for all tested SNR conditions, the approximation error for both port configurations exhibits a consistent monotonic decrease as the number of Taylor series expansion terms increases, confirming the convergence of our approach. Moreover, a systematic downward shift in the error curves is observed as the SNR increases from 10 dB to 20 dB, indicating that better channel conditions allow the desired accuracy to be achieved with fewer expansion terms, thereby improving computational efficiency. In this study, we adopt a threshold of $\varepsilon_{\text{rel}} = 10^{-3}$ for evaluating the accuracy of the proposed closed-form expression. This choice is grounded in rigorous engineering judgment: when the reference outage probability approaches the ultra-reliable communication target of 99.999%, this threshold ensures that the absolute approximation error does not exceed about 10^{-3} . Such an error margin is considerably smaller than typical engineering safety allowances and therefore does not compromise the core conclusion regarding whether the system meets the 99.999% reliability target. At the same time, this threshold strikes a balance between numerical rigor and practical utility. Fig. 4 illustrates how the relative error varies with SNR.

The simulation results confirm the effectiveness and conver-

gence of the proposed Taylor-series-based approximation. The approximation error decreases monotonically as the number of retained terms increases, and becomes negligible in the medium-to-high SNR regime.

Based on the above analysis, the system outage probability for different numbers of expansion terms ($M = 2, 4, 6$) is evaluated against the increasing SNR, as shown in Fig. 5.

The proposed approximations achieve remarkable accuracy with substantially reduced computational cost compared to the full model. The 4-term and 6-term expansions show excellent agreement with the reference model across the entire SNR range, outperforming the 2-term approximation, especially in the low-SNR regime. In medium to high SNR regions (approximately above 10 dB), both approximations consistently satisfy the ultra-reliable communication criterion of $P_{\text{out}} < 10^{-5}$ across the entire SNR range. Moreover, the consistent align-

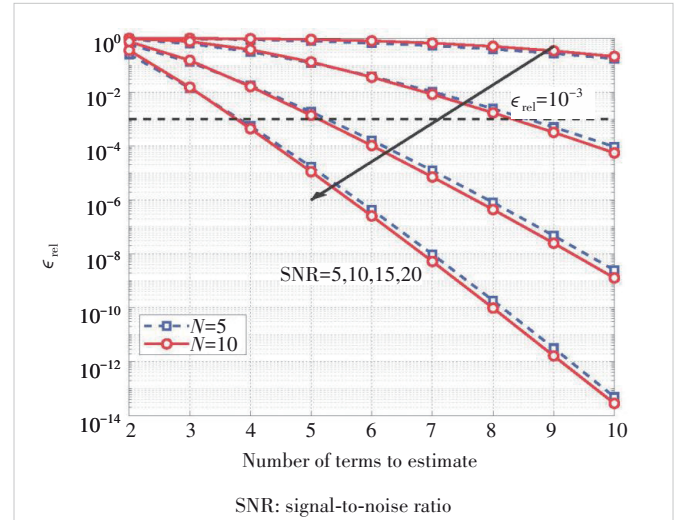


Figure 3. ε_{rel} versus the number of series expansion terms for I_0

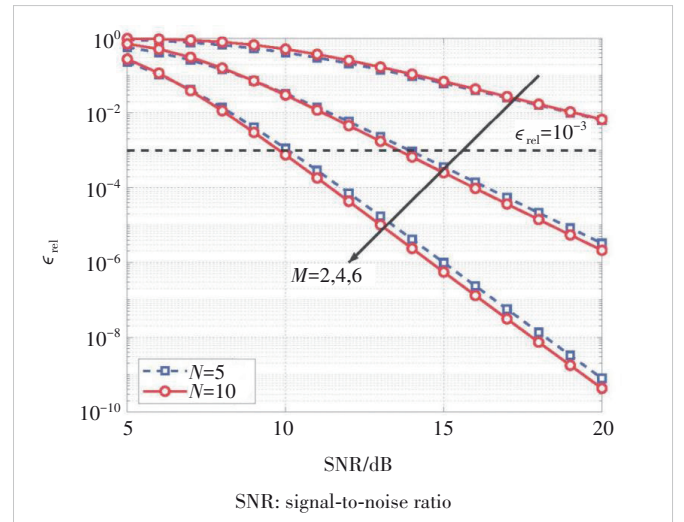


Figure 4. ε_{rel} versus SNR

ment across different port configurations ($N = 5$ and $N = 10$) further validates the adaptability and scalability of the proposed method.

5 Conclusions

In this paper, we tackle the challenging problem of deriving a closed-form expression for the outage probability of an FAS under the practical block-correlation model. The primary obstacle in this endeavor lies in the analytical intractability of the Marcum Q-function embedded within the outage probability integral. To overcome this challenge, our first key contribution is the introduction of a novel approximation for the Bessel function I_0 . By applying a precise M -term Taylor series expansion directly to its integral kernel, we successfully reformulated this function into a manageable M -term series representation. Building upon this mathematical foundation, our second major contribution was the successful derivation of an approximate closed-form expression for the FAS outage probability. By leveraging multinomial theorem and other analytical techniques, we consolidated the derived series into a comprehensive and computationally efficient formula.

Furthermore, the proposed approximation method for the Marcum Q-function is not limited to the block-correlation model and the Rayleigh channel. The core mathematical framework is generally applicable to both alternative spatial correlation models and broader classes of spatial fading models for FAS². Future work will focus on extending this analysis to multi-user FAS scenarios and incorporating more sophisticated fading channel models.

Appendix A

From (10), we can obtain the expression of $P_{\text{out}}(\gamma)$ like

$$P_{\text{out}}(\gamma) = \prod_{b=1}^B \int_0^\infty \frac{1}{2^{L(b)+1}} \exp\left(-r_0 \left(\frac{1}{2} + \frac{\mu^2 L(b)}{2(1-\mu^2)}\right)\right) \times \prod_{k=1}^{L(b)} \left[\sum_{m=0}^{M-1} \frac{1}{4^m (m!)^2} \left(\frac{\mu^2 r_0}{1-\mu^2}\right)^m \int_0^{\frac{\gamma}{1-\mu^2}} e^{-\frac{t_k}{2}} t_k^m dt_k \right] dr_0 \quad (24).$$

Then, using the lower incomplete Gamma function $\gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} dt$, Eq. (24) becomes

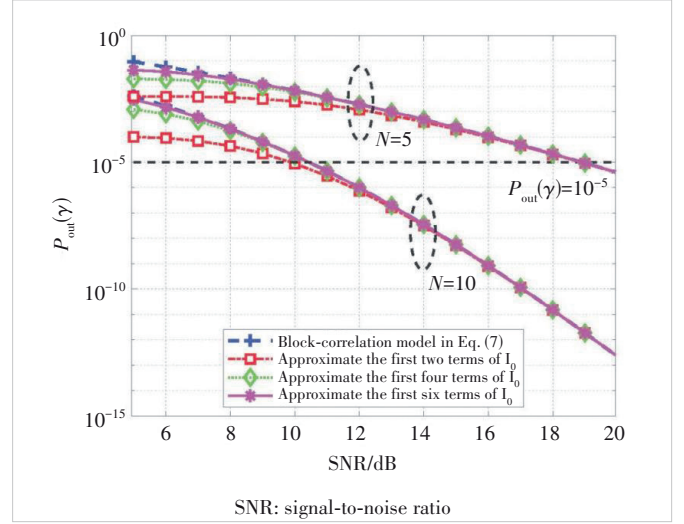


Figure 5. Outage probability versus SNR

$$P_{\text{out}}(\gamma) = \prod_{b=1}^B \int_0^\infty \frac{1}{2^{L(b)+1}} e^{-\alpha r_0} \left[\sum_{m=0}^{M-1} d_m r_0^m \right]^{L(b)} dr_0 \quad (25),$$

where d_m is given by Eq. (17).

To expand $\left[\sum_{m=0}^{M-1} d_m r_0^m \right]^{L(b)}$, we employ the multinomial expansion of a polynomial power, i. e. ,

$$\left[\sum_{m=0}^{M-1} d_m r_0^m \right]^{L(b)} = \sum_{k_0 + k_1 + \dots + k_M = L(b)} \binom{L(b)}{k_0, k_1, \dots, k_M} \times \prod_{j=0}^{M-1} d_j^{k_j} r_0^{\sum_{j=0}^{M-1} j k_j} \quad (26).$$

Then we can get

$$P_{\text{out}}(\gamma) = \prod_{b=1}^B \frac{1}{2^{L(b)+1}} \sum_{k_0 + k_1 + \dots + k_M = L(b)} \binom{L(b)}{k_0, k_1, \dots, k_M} \times \prod_{j=0}^{M-1} d_j^{k_j} \int_0^\infty e^{-\alpha r_0} r_0^s dr_0 \quad (27),$$

where s is given by Eq. (17). Using the Gamma function, we can get the final result like Eq. (16).

Appendix B

B.1 Proof of Theorem 1

Since the Taylor series of $I_0(x)$ is a positive series and its coefficient $a_k = 1/(k!)^2$ decays rapidly as k increases, it fol-

² This generality exists because the performance analysis of FAS under both alternative spatial correlation models (e.g., Jakes' model) and various fading models (e.g., Rician, Nakagami- m) consistently leads to probability expressions whose integral kernels involve special functions such as Bessel functions, Marcum Q-functions, or their variants^[14-15]. Precisely because the performance analysis under these different models ultimately yields integral expressions with similar kernels, the method proposed in our study can provide a unified processing strategy to transform such integrals into analytically tractable polynomial forms.

lows that:

$$\begin{aligned} \varepsilon_{I_0}(x, M) &= \sum_{k=M}^{\infty} \frac{x^{2k}}{2^{2k}(k!)^2} \leq \frac{x^{2M}}{2^{2M}(M!)^2} \sum_{k=0}^{\infty} \frac{x^{2k}}{2^{2k}(k!)^2} = \\ &= \frac{x^{2M}}{2^{2M}(M!)^2} I_0(x) \end{aligned} \quad (28).$$

B.2 Proof of Theorem 2

We substitute the error bound from Theorem 1 into the outage integral, interchange integral and summation, and bound the resulting expression using the exponential decay. Specific steps are as follows:

$$\begin{aligned} \varepsilon_{P_{out}} &= \left| \int_0^{\infty} e^{-2t} \varepsilon_{I_0}(2\sqrt{Ft}, M) dt \right| \leq \\ &= \int_0^{\infty} e^{-2t} \cdot \varepsilon_{I_0}(2\sqrt{Ft}, M) dt \leq \\ &= \int_0^{\infty} e^{-2t} \cdot \frac{(\sqrt{Ft})^{2M}}{(M!)^2} I_0(2\sqrt{Ft}) dt = \\ &= \frac{F^M}{(M!)^2} \int_0^{\infty} e^{-2t} t^M I_0(2\sqrt{Ft}) dt \end{aligned} \quad (29).$$

We define $C(M, F) = \int_0^{\infty} e^{-2t} t^M I_0(2\sqrt{Ft}) dt$ and then prove that it is bounded. The integrand is continuous for $t \geq 0$. As $t \rightarrow \infty$, we use the asymptotic expansion for I_0 :

$$I_0(2\sqrt{pt}) \sim \frac{e^{2\sqrt{pt}}}{\sqrt{4\pi\sqrt{pt}}} \quad (30).$$

So we obtain

$$e^{-2t} \cdot I_0(2\sqrt{Ft}) \sim \frac{e^{-2t+2\sqrt{Ft}}}{\sqrt{4\pi\sqrt{Ft}}} \quad (31).$$

The exponential term $-2t + \sqrt{2Ft}$ is dominated by $-2t$. For large t , the integrand decays exponentially, which results in overall exponential decay and ensures the convergence of the integral. Hence, $C(M, F)$ is bounded, completing the proof of Theorem 2.

B.3 Proof of Theorem 3

We define the following sequence of functions:

$$g_M(t) = e^{-2t} T_M(2\sqrt{Ft}) \quad (32).$$

Since I_0 is an entire function, its Taylor series converges uniformly on the entire complex plane, and therefore it also converges pointwise on the real axis. Hence, for any fixed $t \geq 0$ and $F \geq 0$, we have $\lim_{M \rightarrow \infty} T_M(2\sqrt{Ft}) = I_0(2\sqrt{Ft})$.

To apply the Lebesgue dominated convergence theorem, we construct a dominating function $G(t)$. Note that:

$$|g_M(t)| \leq e^{-2t} |T_M(2\sqrt{Ft})| \leq e^{-2t} I_0(2\sqrt{Ft}) = G(t) \quad (33).$$

We now prove that $G(t)$ is integrable on $[0, \infty)$. As $t \rightarrow \infty$,

$$G(t) = e^{-2t} \cdot I_0(2\sqrt{Ft}) \sim \frac{e^{-2t+2\sqrt{Ft}}}{\sqrt{4\pi\sqrt{Ft}}} \quad (34).$$

The exponential term $-2t + \sqrt{2Ft}$ is dominated by $-2t$ for large t , hence $G(t)$ decays exponentially, which guarantees the convergence of the integral. According to the Lebesgue dominated convergence theorem:

$$\lim_{M \rightarrow \infty} \int_0^{\infty} g_M(t) dt = \int_0^{\infty} \lim_{M \rightarrow \infty} g_M(t) dt = \int_0^{\infty} g(t) dt \quad (35).$$

So we obtain

$$\lim_{M \rightarrow \infty} P_{out}^{(M)} = 1 - \lim_{M \rightarrow \infty} \int_0^{\infty} g_M(t) dt = 1 - \int_0^{\infty} g(t) dt = P_{out}^{\text{exact}} \quad (36).$$

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