Joint Beamforming Design for Dual-Functional Radar-Communication Systems Under Beampattern Gain Constraints

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DOI: 10.12142/ZTECOM.202403003

https://kns.cnki.net/kcms/detail/34.1294.TN.20240814.1632.002.html, published online August 15, 2024

Manuscript received: 2024-06-22

Abstract: The joint beamforming design challenge for dual-functional radar-communication systems is addressed in this paper. The base station in these systems is tasked with simultaneously sending shared signals for both multi-user communication and target sensing. The primary objective is to maximize the sum rate of multi-user communication, while also ensuring sufficient beampattern gain at particular angles that are of interest for sensing, all within the constraints of the transmit power budget. To tackle this complex non-convex problem, an effective algorithm that iteratively optimizes the joint beamformers is developed. This algorithm leverages the techniques of fractional programming and semidefinite relaxation to achieve its goals. The numerical results confirm the effectiveness of the proposed algorithm.

Keywords: dual-functional radar-communication; joint beamforming design; beampattern gain constraints; semidefinite relaxation; fractional programming

Citation (Format 1): CHEN G Y, ZHANG R Y, REN H, et al. Joint beamforming design for dual-functional radar-communication systems under beampattern gain constraints [J]. ZTE Communications, 2024, 22(3): 13 – 20. DOI: 10.12142/ZTECOM.202403003

Citation (Format 2): G. Y. Chen, R. Y. Zhang, H. Ren, et al., "Joint beamforming design for dual-functional radar-communication systems under beampattern gain constraints," *ZTE Communications*, vol. 22, no. 3, pp. 13 – 20, Sept. 2024. doi: 10.12142/ZTECOM.202403003.

1 Introduction

In future 6G wireless networks, we expect that the scarcity of spectrum resources will be exacerbated by the increasing number of wireless communication devices and higher demands for transmission rates[1 – 3]. The growing sensing requirements in applications such as unmanned aerial vehicles and intelligent vehicles have made the coexistence of radar-communication spectrum a vital issue[4 – 6]. Due to the benefits of low hardware complexity reduction, spectrum sharing, low power consumption, and joint signal processing, dual-functional radar-communication (DFRC) is now regarded as a key enabling technology in 6G systems[7 – 8].

In the DFRC systems where radar and communication share a platform, the joint beamforming design enables multi-user communication and radar sensing by exploiting the spatial degree of freedom (DoF)[9]. In Ref. [10], the goal was to minimize the radar beampattern mean square error while satisfying communication quality of service constraints. In Ref. [11], the beamforming design was proposed to maximize the worst signal-to-interference-noise ratio (SINR) among all users, while satisfying the transmit waveform covariance and power constraints. The Cramér-Rao bound was used as the radar performance metric and the SINR as the communication metric to optimize beamformers[12]. Under constraints of power and signal-clutter-noise ratio, a low-complexity beamforming scheme to maximize the sum rate was investigated in Ref. [13].

While these works considered the communication metrics as the objective function, the authors investigated the joint beamforming design using the radar metrics as the objective function in Refs. [14 – 16]. In Ref. [14], an approach to minimizing the radar beampattern mean squared error under the SINR constraints was proposed. In Ref. [15], under the same SINR constraint, the authors developed the joint beamformer by matching the radar detection beampattern. In Ref. [16], the authors compared the beamforming designs of the beampat-

This work was supported in part by the National Natural Science Foundation of China under Grant No. 62201266, and in part by the Natural Science Foundation of Jiangsu Province under Grant No. BK20210335.

tern matching error minimization and the beampattern gain maximization under the power and SINR constraints. A shortcoming of the conventional beampattern matching design is that a fine grid of points covering the location sectors of interest is required to approximate the desired beampattern. In contrast, the beampattern gain maximization design focuses on the direct optimizing of the radar direction gain without requiring complex dense grids, which inspires us to leverage this characteristic for DFRC beamforming design.

In this work, we study the joint beamforming design problem for the DFRC systems in which a base station (BS) transmits the shared signals for both multi-user communication and radar target sensing. Our goal is to maximize the sum rate under the constraints of the radar beampattern gain and the transmit power budget. To tackle the non-convexity of the problem at hand, we employ the fractional programming (FP) method to obtain a tractable form of the objective function[17]. Additionally, the non-convex radar beampattern gain constraints are handled using the semidefinite relaxation (SDR) technique[18]. By doing so, we design an iterative algorithm to obtain the joint beamformers for radar sensing and multi-user communication. Numerical results demonstrate that a flexible trade-off between the communication sum rate and radar beampattern gain performance can be achieved by the proposed algorithm.

Notations are as follows: $\boldsymbol{A}$ and $\boldsymbol{a}$ denote a matrix and a column vector; Superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ denote the matrix transpose, the conjugate and the conjugate transpose, respectively. Expectation and the real part of a complex variable are denoted by $\mathbb{E}\{\cdot\}$ and $\Re\{\cdot\}$; $\mathrm{Tr}(\cdot)$ stands for the trace of a matrix. $\boldsymbol{I}_N$ is an $N \times N$ identity matrix; $\mathrm{rank}(\boldsymbol{A})$ denotes the rank of $\boldsymbol{A}$, and $\boldsymbol{A} \succcurlyeq 0$ indicates that $\boldsymbol{A}$ is positive semidefinite.

2 System Model and Problem Formulation

As shown in Fig. 1, we consider a DFRC system in which the BS is equipped with an N-antenna uniform linear array (ULA). The BS simultaneously serves K single-antenna users and senses Q potential targets. The shared transmit signal from the BS can be expressed as

$$\boldsymbol{x} = \sum_{k=1}^{K} \boldsymbol{w}_k s_k , \tag{1}$$

where $\boldsymbol{w}_k \in \mathbb{C}^{N \times 1}$ is the beamforming vector for the k-th user, and s_k is the transmitted data symbol satisfying $\mathbb{E}\{s_k s_k^*\} = 1$ and $\mathbb{E}\{s_i s_j^*\} = 0$, $\forall i \neq j$. The power constraint at the BS is $\sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \leqslant P_{\max}$, where $P_{\max}$ is the maximum transmit power budget. The received signal of the k-th communication user can be given by

$$y_k = \boldsymbol{h}_k^H \boldsymbol{x} + n_k,\ \forall k, \tag{2}$$

where $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ is the independent identically distributed (i.i.d) complex Gaussian noise with the variance σ_k^2. In this work, we consider the widely adopted geometric channel model given by Ref. [19][1]:

$$\boldsymbol{h}_k = \sqrt{\frac{N}{L_k}} \sum_{l=1}^{L_k} \alpha_{k,l} \boldsymbol{a}(\phi_{k,l}) \in \mathbb{C}^{N \times 1},\ \forall k, \tag{3}$$

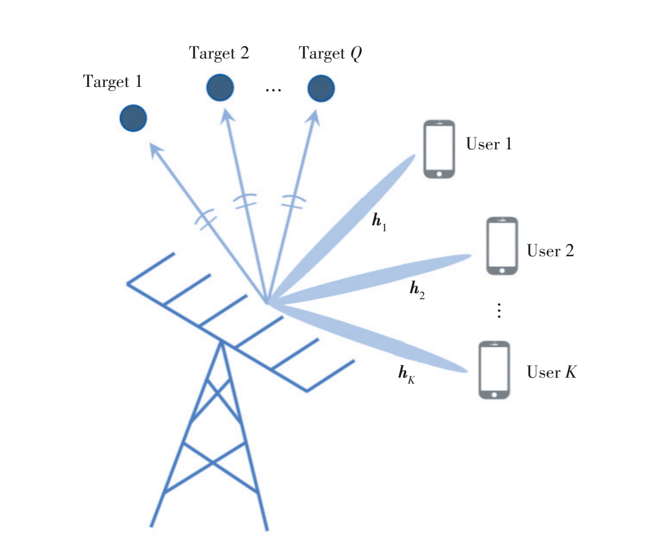
where L_k is the number of paths between the BS and the k-th user, $\alpha_{k,l}$ is the gain of the l-th path for the k-th user, $\phi_{k,l}$ is the angle of departure (AoD) of the k-th user of the l-th path. The transmit steering vector of direction θ is specified as

$$\boldsymbol{a}(\theta) = \frac{1}{\sqrt{N}} \left[1, e^{j\frac{2\pi}{\lambda} d\cos\theta}, \cdots, e^{j\frac{2\pi}{\lambda} d(N-1)\cos\theta}\right]^T, \tag{4}$$

where d and λ are the antenna spacing and signal wavelength. Under this setup, the sum rate of all k users is expressed as

$$R = B \sum_{k=1}^{K} \log_2(1 + \gamma_k), \tag{5}$$

where B is a constant denoting the channel bandwidth. Besides, γ_k is the SINR of the k-th user and is given by



▲ **Figure 1. Illustration of the considered dual-functional radar-communication (DFRC) system**

1 The line-of-sight component is part of the channel model, making it easier to observe the impact of the user's direction on the beam gain. The proposed method is applicable to various types of channels.

$$\gamma_k = \frac{|\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum_{i \neq k}^{K} \left| \boldsymbol{h}_k^H \boldsymbol{w}_i \right|^2 + \sigma_k^2}, \ \forall k \quad (6)$$

For the radar sensing, the detection performance of potential targets can be enhanced by forming highly directional beams towards the target directions. The transmit beampattern gain at a particular angle is given by Ref. [20] as

$$p(\theta) = \mathbb{E}\left\{|\boldsymbol{a}^H(\theta)\boldsymbol{x}|^2\right\} = \boldsymbol{a}^H(\theta)\left(\boldsymbol{w}_k \boldsymbol{w}_k^H\right)\boldsymbol{a}(\theta). \quad (7)$$

Our goal in this work is to design the beamforming vectors $\{\boldsymbol{w}_k\}_{k=1}^K$, such that the sum rate is maximized under the constraints of the transmit power budget and the radar beampattern gain of Q directions is guaranteed. The corresponding optimization problem can be formulated as

$$\max_{\{\boldsymbol{w}_k\}} \sum_{k=1}^{K} \log_2\left(1 + \gamma_k\right), \quad (8a)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \leqslant P_{\max}, \quad (8b)$$

$$\boldsymbol{a}^H\left(\theta_q\right)\left(\sum_{k=1}^{K} \boldsymbol{w}_k \boldsymbol{w}_k^H\right)\boldsymbol{a}\left(\theta_q\right) \geqslant \Gamma_q P_{\max}, \forall q, \quad (8c)$$

where θ_q is the direction of the q-th target, $\Gamma_q P_{\max}$ represents the required beampattern gain towards the q-th target, Γ_q is a weighting coefficient satisfying $\Gamma_1 + \cdots + \Gamma_Q \leqslant 1$. It should be noted that due to the logarithmic and fractional terms in Eq. (8a) and the non-convex radar beampattern gain constraints in Eq. (8c), it is challenging to directly handle the optimization problem.

3 Proposed Joint Beamforming Design for DFRC Systems

In this section, we present an iterative algorithm for solving the considered optimization problem (8). We first use the FP method to tackle the complex objective function (8a) and then transform a considered problem into the tractable form based on the SDR technique.

3.1 Transformation of Objective Function

We study the properties of the objective function (8a), a typical function with multiple fractional terms. Our goal is to convert the objective function to a tractable form. Using the Lagrangian duality transformation[17], we take the fractional term γ_k out of the logarithm and then transform the function (8a) into a polynomial expression.

Proposition 1: The objective function (8a) can be converted into

$$\sum_{k=1}^{K} \log_2\left(1 + \nu_k\right) - \nu_k + \sum_{k=1}^{K} \frac{\left(1 + \nu_k\right)\left|\boldsymbol{h}_k^H \boldsymbol{w}_k\right|^2}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2}, \quad (9)$$

where ν_k is an auxiliary variable satisfying $\nu_k = \gamma_k, \forall k$.

Proof: See Appendix A.

Proposition 1 illustrates that the objective function (8a) is equivalent to Eq. (9) as long as $\nu_k = \gamma_k$. Even after this transform, solving the problem remains challenging due to the last term in the objective function (9), which is the sum of k fractional terms. To address this issue, we use the multidimensional quadratic transform[17].

Proposition 2: The fractional term in Eq. (9), that is

$$\frac{\left(1 + \nu_k\right)|\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2}, \ \forall k, \quad (10)$$

which can be quadratically transformed into

$$2\sqrt{1 + \nu_k}\, \Re\left\{\tau_k^* \boldsymbol{h}_k^H \boldsymbol{w}_k\right\} - \sum_{i=1}^{K} |\tau_i|^2 |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 - |\tau_k|^2 \sigma_k^2, \quad (11)$$

where $\{\tau_k\}_{k=1}^K$ is the fractional programming auxiliary variable expressed as

$$\tau_k = \frac{\sqrt{1 + \nu_k}\, \boldsymbol{h}_k^H \boldsymbol{w}_k}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2}, \ \forall k. \quad (12)$$

Proof: See Appendix B.

Using Proposition 2, Eq. (9) can be further reformulated as

$$\sum_{k=1}^{K} \log\left(1 + \nu_k\right) - \sum_{k=1}^{K} \nu_k + \sum_{k=1}^{K}\left(2\sqrt{1 + \nu_k}\, \Re\left\{\tau_k^* \boldsymbol{h}_k^H \boldsymbol{w}_k\right\} - \sum_{i=1}^{K} |\tau_i|^2 |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 - |\tau_k|^2 \sigma_k^2\right). \quad (13)$$

To facilitate the beamforming design, we determine the optimal auxiliary variables ν_k and τ_k by applying $\nu_k = \gamma_k$ and Eq. (12). Then, we extract the term containing $\boldsymbol{w}_k$ from the objective function (13) and rewrite the objective function as

$$\sum_{k=1}^{K}\left(2\sqrt{1 + \nu_k}\, \Re\left\{\tau_k^* \boldsymbol{h}_k^H \boldsymbol{w}_k\right\} - \sum_{i=1}^{K} |\tau_i|^2 |\boldsymbol{h}_i^H \boldsymbol{w}_k|^2\right) = \sum_{k=1}^{K}\left(\sqrt{1 + \nu_k}\left(\tau_k^* \boldsymbol{h}_k^H \boldsymbol{w}_k + \boldsymbol{w}_k^H \boldsymbol{h}_k \tau_k\right) - \boldsymbol{w}_k^H \sum_{i=1}^{K} |\tau_i|^2 \boldsymbol{h}_i \boldsymbol{h}_i^H \boldsymbol{w}_k\right), \quad (14)$$

where the equality holds due to $2\Re\{\boldsymbol{A}\} = \boldsymbol{A} + \boldsymbol{A}^H$. By defining $\boldsymbol{\beta}_k = \sqrt{1+\nu_k}\,\boldsymbol{h}_k\tau_k$ and $\Lambda = \sum_{i=1}^{K}|\tau_i|^2\boldsymbol{h}_i\boldsymbol{h}_i^H$, the optimization problem can be compactly formulated as

$$\max_{\{\boldsymbol{w}_k\}} \sum_{k=1}^{K}\left(\boldsymbol{\beta}_k^H\boldsymbol{w}_k + \boldsymbol{w}_k^H\boldsymbol{\beta}_k - \boldsymbol{w}_k^H\boldsymbol{\Lambda}\boldsymbol{w}_k\right), \tag{15a}$$

$$\text{s.t.} \sum_{k=1}^{K}|\boldsymbol{w}_k|^2 \leqslant P_{\max}, \tag{15b}$$

$$\boldsymbol{a}^H(\theta_q)\left(\sum_{k=1}^{K}\boldsymbol{w}_k\boldsymbol{w}_k^H\right)\boldsymbol{a}(\theta_q) \geqslant \Gamma_q P_{\max}, \forall q. \tag{15c}$$

One can see that (15) is a non-homogeneous quadratic constrained quadratic programming (QCQP) problem. In the next subsection, we reformulate the problem (15) as a homogeneous QCQP problem and adopt the SDR technique to obtain the optimized DFRC beamformers.

3.2 Solution via SDR

We first derive the equivalent form of beampattern gain constraint (15c) by introducing the auxiliary variable t, i.e.,

$$\sum_{k=1}^{K}\boldsymbol{a}^H(\theta_q)\left(\sum_{i=1}^{K}\boldsymbol{w}_k\boldsymbol{w}_k^H\right)\boldsymbol{a}(\theta_q) = \sum_{k=1}^{K}\left[\boldsymbol{a}^H(\theta_q), 0\right]\begin{bmatrix}\boldsymbol{w}_k\\ t\end{bmatrix}\left[\boldsymbol{w}_k^H, t\right]\begin{bmatrix}\boldsymbol{a}(\theta_q)\\ 0\end{bmatrix} = \sum_{i=1}^{K}\boldsymbol{x}_k^H\boldsymbol{C}_q\boldsymbol{x}_k, \tag{16}$$

where the last equality holds by defining

$$\boldsymbol{x}_k = \begin{bmatrix}\boldsymbol{w}_k\\ t\end{bmatrix}, \boldsymbol{C}_q = \begin{bmatrix}\boldsymbol{a}(\theta_q)\boldsymbol{a}(\theta_q)^H & 0\\ 0 & 0\end{bmatrix}, \boldsymbol{X}_k = \boldsymbol{x}_k\boldsymbol{x}_k^H. \tag{17}$$

The non-homogeneous objection function (15a) can be equivalently expressed as the following homogeneous form:

$$\sum_{k=1}^{K}\left(\left[\boldsymbol{w}_k^H, t\right]\begin{bmatrix}-\boldsymbol{\Lambda} & \boldsymbol{\beta}_k\\ \boldsymbol{\beta}_k^H & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{w}_k\\ t\end{bmatrix}\right), \tag{18}$$

where Eq. (18) equals to the objective function (15a) by introducing the constraint $t^2 = 1$, which can be equivalently written by

$$t^2 = \left[\boldsymbol{w}_k^H, t\right]\begin{bmatrix}0_N & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\boldsymbol{w}_k\\ t\end{bmatrix} = 1. \tag{19}$$

Also, the power constraint (15b) can be manipulated as

$$\sum_{k=1}^{K}\left[\boldsymbol{w}_k^H, t\right]\begin{bmatrix}\boldsymbol{I}_N & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{w}_k\\ t\end{bmatrix} \leqslant P_{\max}. \tag{20}$$

For simplicity, we define

$$\boldsymbol{A}_k = \begin{bmatrix}-\Lambda & \boldsymbol{\beta}_k\\ \boldsymbol{\beta}_k^H & 0\end{bmatrix}, \boldsymbol{B} = \begin{bmatrix}\boldsymbol{I}_N & 0\\ 0 & 0\end{bmatrix}, \boldsymbol{D} = \begin{bmatrix}0_N & 0\\ 0 & 1\end{bmatrix}. \tag{21}$$

Optimization problem (15) can be restated as

$$\begin{aligned}&\max_{\{\boldsymbol{X}_k\}\in\mathbb{H}^{N+1}} \sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{A}_k\boldsymbol{X}_k\right),\\ &\text{s.t.} \sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{B}\boldsymbol{X}_k\right) \leqslant P_{\max},\\ &\sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{C}_q\boldsymbol{X}_k\right) \geqslant \Gamma_q P_{\max}, \forall q,\\ &\mathrm{Tr}\left(\boldsymbol{D}\boldsymbol{X}_k\right) = 1, \mathrm{rank}\left(\boldsymbol{X}_k\right) = 1,\\ &\boldsymbol{X}_k \succcurlyeq 0,\end{aligned} \tag{22}$$

where $\mathbb{H}^{N+1}$ is the set of $N+1$ dimensional complex Hermitian matrices. Note that Eq. (22) is still a non-convex optimization problem due to the existence of rank-one constraints. We relax the non-convex constrains and then transform problem (22) into a convex one. Then, we can solve the converted problem using the CVX toolbox in MATLAB[21]. Finally, by applying the Gaussian randomization technique to reduce the rank of the $\boldsymbol{X}_k$ matrix to one[18], we obtain the beamforming vectors $\{\boldsymbol{w}_k\}_{k=1}^{K}$. Algorithm 1 summarizes the proposed algorithm for problem (8).

Algorithm 1. The Proposed Algorithm for Problem (8)

Input: $N, K, \boldsymbol{h}_1, \cdots, \boldsymbol{h}_K$, maximum iteration number $\text{iter}_{\max}$, and threshold $\varepsilon > 0$

Output: $\{\boldsymbol{w}_k\}_{k=1}^{K}$

1. Initialize randomly $\{\boldsymbol{w}_k^{(0)}\}_{k=1}^{K}$, compute the sum rate $R^{(0)}$;
2. **While** iter $\leqslant \text{iter}_{\max}$ and $\left|R^{(\text{iter})} - R^{(\text{iter}-1)}\right|/B \geqslant \varepsilon$ **do**
3. Update $\{\nu_k^{(\text{iter})}\}_{k=1}^{K}$ via $\nu_k = \gamma_k, \forall k$;
4. Update $\{\tau_k^{(\text{iter})}\}_{k=1}^{K}$ via (12);
5. Update $\{\boldsymbol{w}_k^{(\text{iter})}\}_{k=1}^{K}$ via (22) and Gaussian randomization technique;
6. Compute $R^{(\text{iter})}$ via (5);
7. iter=iter+1;
8. **End while.**

The main computational complexity of the overall algorithm is dominated by Step 5 of Algorithm 1. For solving problem (22), the interior-point method is commonly utilized[12]. The computational complexity of updating $\{\boldsymbol{X}_k\}_{k=1}^{K}$ is $\mathcal{O}\left(K^{3.5}N^7\log\left(1/\epsilon\right)\right)$, where ϵ is the given accuracy level. The Gaussian randomization technique is then employed to recover the beamforming vectors, which adds a complexity of

$\mathcal{O}\left(K^{2.5}N^{3}\right)$. The overall complexity is of order $\mathcal{O}\left(K^{3.5}N^{7}\log\left(1/\epsilon\right)+\mathcal{O}\left(K^{2.5}N^{3}\right)\right)$[22].

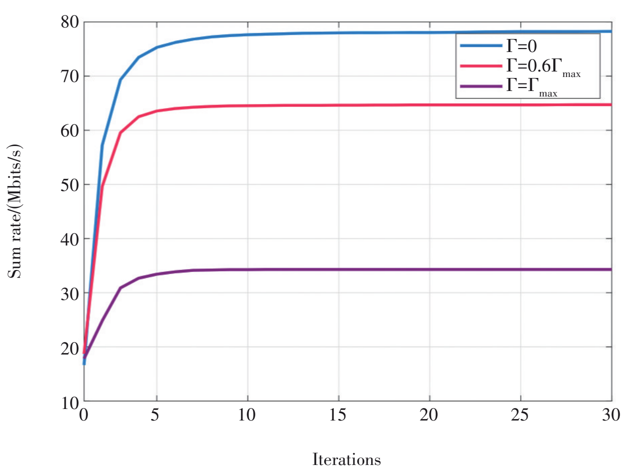
4 Numerical Results

In this section, we present the numerical results to investigate the performance of the proposed DFRC joint beamforming design.

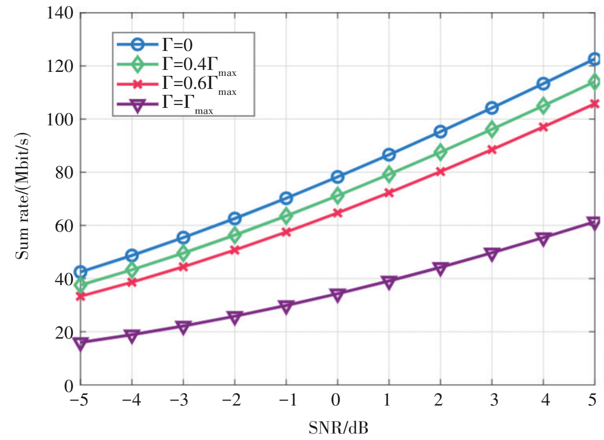
Unless stated otherwise, we assume that the BS equipped with $N=16$ antennas is serving $K=4$ users[23], and the antenna spacing is set to $d=\lambda/2$. As for the channel model, we consider $L_k=5$ paths with $\alpha_{k,l}\sim\mathcal{CN}\left(0,1\right)$. The AoDs follow the Laplacian distribution with uniformly distributed in $[0,\pi)$ and angular spread of five degrees [24]. We set the bandwidth as $B=10$ MHz. In addition, we set $Q=3$ sensing directions with angles of 50°, 90° and 130°[13]. We assume that the beampattern gain is the same for all target directions $\left(\Gamma_q=\Gamma,\forall q\right)$, which allows us to calculate the maximum weighting coefficient of beampattern gain for each direction as $\Gamma_{\max}=1/Q$. The signal-to-noise ratio (SNR) is defined as $P_{\max}/\sigma_k^2$, with $\sigma_k^2=1$, $\forall k$. We set the maximum iteration number $\text{iter}_{\max}=100$ and threshold $\varepsilon=10^{-3}$. The performance of the proposed algorithm is averaged over 500 Monte-Carlo realizations.

In Fig. 2, we demonstrate the convergence behavior of the proposed algorithm under different beampattern gains, while holding a fixed SNR of 0 dB. It is evident that the proposed algorithm converges after 30 iterations. This trend is steady, indicating a robust and stable algorithm.

Fig. 3 illustrates the performance of sum rates versus SNR for different values of Γ. As seen in Fig. 3, the sum rate is the highest in the communication-only case ($\Gamma=0$), which means there is no constraint imposed by the radar beampattern gain. Conversely, as Γ incrementally increases, a noticeable decline in the sum rate performance is observed. This trend can be attributed to the shifting design focus towards enhancing radar sensing capabilities. In essence, as the value of Γ escalates, the system's priority transitions from solely maximizing communication sum rate to a more balanced approach.



▲Figure 2. Sum rate convergence behavior of the proposed algorithm

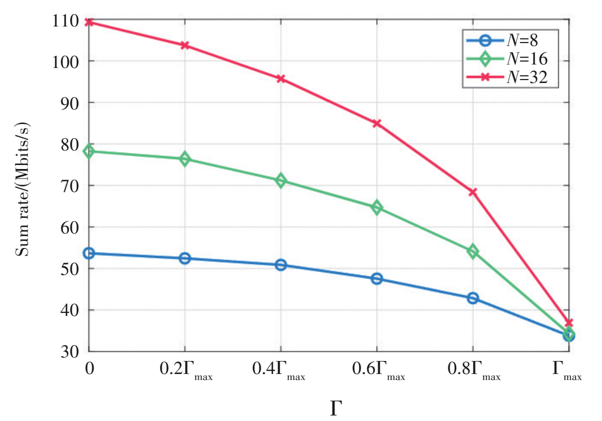


SNR: signal-to-noise ratio

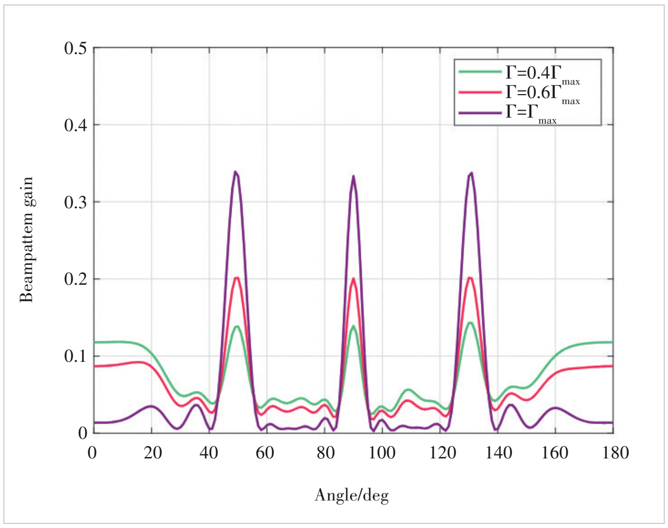
▲Figure 3. Sum rate versus SNR for different Γ

We evaluate the trade-off between the communication sum rate and the radar beampattern gain with different numbers of BS antennas in Fig. 4, where SNR is set to 0 dB. It can be seen that the sum rate performance increases with the number of antennas. Additionally, when the number of antennas increases, the beampattern gain increases as well due to the increased DoF.

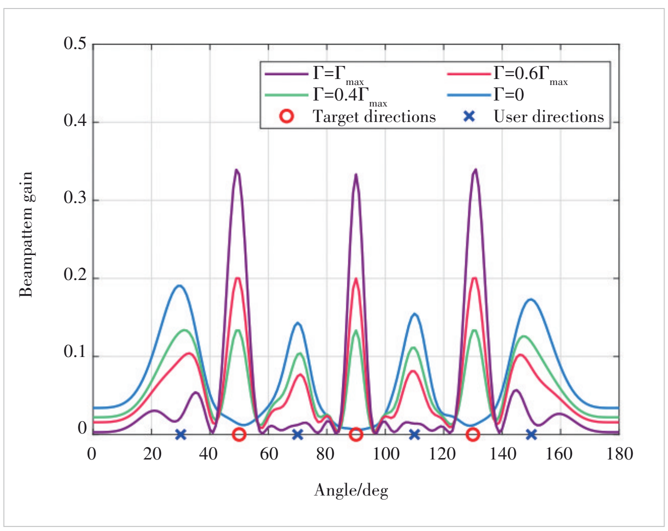
Fig. 5 depicts the beampattern gain of the proposed design when SNR = 0 dB. Our proposed DFRC design simultaneously allocates the beampattern gain to the directions of the sensing targets and communication users. Specifically, as the value of Γ increases, the beampattern gain weight becomes more tilted towards the radar, which is consistent with the results in Fig. 3, where the sum rate gradually decreases. In Fig. 6, we further present the beampattern performance, where users are located at $\left[30^\circ,70^\circ,110^\circ,150^\circ\right]$. One can clearly ob-



▲Figure 4. Sum rate versus Γ for different numbers of antenna



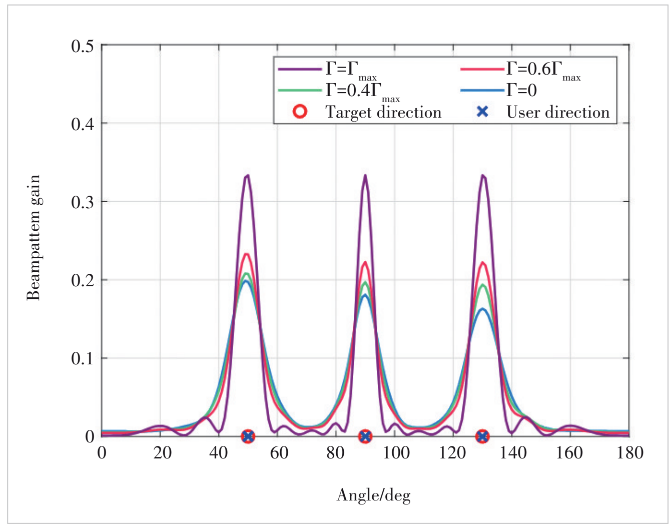
▲**Figure 5. Radar sensing beampattern performance comparison of different Γ, where user directions are uniformly distributed from $[0, \pi)$**



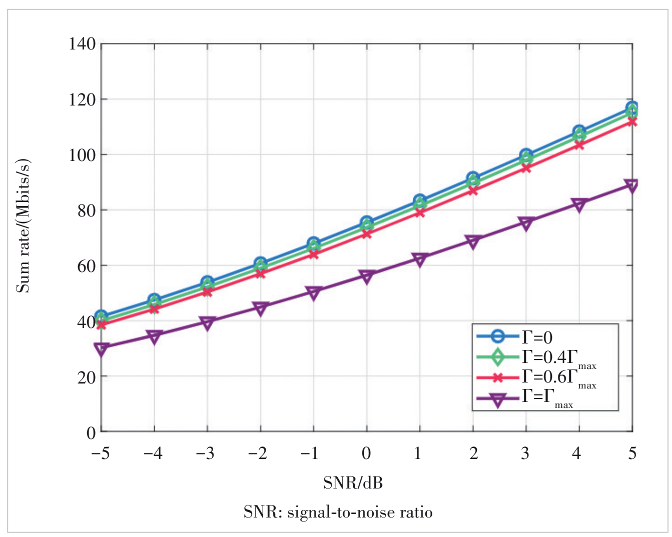
▲**Figure 6. Radar sensing beampattern performance comparison of different Γ, where users are located at [30°, 70°, 110°, 150°]**

serve that the beampattern gain is allocated to the desired directions of the sensing targets and users in the radar-only ($\Gamma = \Gamma_{max}$) and communication-only ($\Gamma = 0$) cases.

Fig. 7 depicts the radar sensing beampattern performance for different values when the positions of the communication users and radar targets coincide. One can see that, as the Γ value increases, the beampattern gain at the radar target and communication target directions (50°, 90°, 130°) increases. In addition, compared to the case where users and targets are located at different directions in Figs. 5 and 6, the case where users and radar targets coincide shows higher beampattern gain at the same value of Γ. Fig. 8 shows that the proposed method achieves a balance between communication and sensing. In the radar-only case, compared to the scenario where the communication users and radar targets are separated, the



▲**Figure 7. Radar sensing beampattern performance comparison of different Γ, where the positions of the communication users and radar targets coincide**



▲**Figure 8. Sum rate versus SNR for different Γ, where the positions of the communication users and radar targets coincide**

sum rate performance is better.

5 Conclusions

In this work, we study a joint beamforming design problem for the DFRC system and propose an iterative algorithm to maximize the sum rate under the radar beampattern gain and power constraints. Utilizing the FP technique, we transform the complex non-convex problem into a more tractable form and apply the SDR technique to solve this transformed problem. From our experimentation, we demonstrate that the proposed algorithm can achieve a flexible trade-off between the communication sum rate and the radar beampattern gain performance.

Appendix A

By introducing K auxiliary variables ν_k to replace each fractional term γ_k in Eq. (5), we can rewrite the unconstrained sum rate maximization problem as

$$\max \sum_{k=1}^{K} \log\left(1+\nu_k\right) \quad \text{s.t. } \nu_k \leqslant \gamma_k, \forall k. \tag{23}$$

Note that Eq. (23) is a convex optimization problem satisfying the strong duality[25]. By introducing K multipliers λ_k, we can obtain a Lagrangian function as

$$\mathcal{L} = \sum_{k=1}^{K} \log\left(1+\nu_k\right) - \sum_{k=1}^{K} \lambda_k\left(\nu_k - \gamma_k\right). \tag{24}$$

By setting $\partial\mathcal{L}/\partial\nu_k = 0$, we obtain the optimal $\nu_k^\star = \gamma_k$. By setting $\partial\mathcal{L}/\partial\lambda_k = 0$, we can obtain the optimal $\lambda_k^\star$ as

$$\lambda_k^\star = \frac{1}{1+\gamma_k}. \tag{25}$$

With the optimal $\lambda_k^\star$, Eq. (24) can be written as

$$\begin{aligned}
\mathcal{L}^\star &= \sum_{k=1}^{K} \log\left(1+\nu_k\right) - \sum_{k=1}^{K} \lambda_k^\star\left(\nu_k - \gamma_k\right) = \\
&\sum_{k=1}^{K} \log\left(1+\nu_k\right) - \sum_{k=1}^{K} \frac{\nu_k}{1+\gamma_k} + \sum_{k=1}^{K} \frac{\gamma_k}{1+\gamma_k} = \\
&\sum_{k=1}^{K} \log\left(1+\nu_k\right) - \sum_{k=1}^{K} \frac{\nu_k\left(\sum_{i\neq k}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2 + |\boldsymbol{h}_k^H \boldsymbol{w}_k|^2\right)}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2} + \\
&\sum_{k=1}^{K} \frac{\nu_k |\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2} + \sum_{k=1}^{K} \frac{|\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2} = \\
&\sum_{k=1}^{K} \log\left(1+\nu_k\right) - \sum_{k=1}^{K} \nu_k + \sum_{k=1}^{K} \frac{\left(1+\nu_k\right) |\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2},
\end{aligned} \tag{26}$$

where the second equation holds by substituting the expression of γ_k in Eq. (6). Expression (26) has the same form as Eq. (9) in Proposition 1. By substituting $\nu_k^\star$ back into Eq. (9), we can obtain objective function (8a), which completes the proof.

Appendix B

To obtain an equivalent form of Eq. (10), we can rewrite Eq. (11) as

$$\begin{aligned}
&\sqrt{1+\nu_k}\,\Re\{\tau_k^* \boldsymbol{h}_k^H \boldsymbol{w}_k\} - \sum_{i=1}^{K} |\tau_i|^2 |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 - |\tau_k|^2 \sigma_k^2 = \\
&\left(\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2\right)\left(\frac{2\sqrt{1+\nu_k}\,\Re\text{big}\{\tau_k^* \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{w}_k \text{big}\}}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2} - |\tau_k|^2\right) = \\
&\left(\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2\right)\left(\frac{\left(1+\nu_k\right)|\boldsymbol{h}_k^H \boldsymbol{w}_k|^2}{\left(\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2\right)^2} - \left|\frac{\sqrt{1+\nu_k}\,\boldsymbol{h}_k^H \boldsymbol{w}_k}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2} - \tau_k\right|^2\right), \forall k.
\end{aligned} \tag{27}$$

where the first equation holds by factoring out the common terms from $\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2$. Note that, when we have

$$\tau_k^\star = \frac{\sqrt{1+\nu_k}\,\boldsymbol{h}_k^H \boldsymbol{w}_k}{\sum_{i=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{w}_i|^2 + \sigma_k^2}, \forall k, \tag{28}$$

Eq. (27) becomes equivalent to Eq. (10) in Proposition 2, which completes the proof.

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