



Design of Raptor-Like Rate Compatible SC-LDPC Codes

Abstract: This paper proposes a family of raptor-like rate-compatible spatially coupled low-density parity-check (RL-RC-SC-LDPC) codes from RL-RC-LDPC block codes. There are two important keys. One is the performance of the base matrix. RL-LDPC codes have been adopted in the technical specification of 5G new radio (5G-NR). We use the 5G NR LDPC code as the base matrix. The other is the edge coupling design. In this regard, we have designed a rate-compatible coupling algorithm, which can improve performance under multiple code rates. The constructed RL-RC-SC-LDPC code property requires a large coupling length L and thus we improved the reciprocal channel approximation (RCA) algorithm and proposed a sliding window RCA algorithm. It can provide lower complexity and latency than RCA algorithm. The code family shows improved thresholds close to the Shannon limit and finite-length performance compared with 5G NR LDPC codes for the additive white Gaussian noise (AWGN) channel.

Keywords: SC-LDPC code; 5G NR LDPC code; rate-compatibility; threshold; sliding window RCA algorithm

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1 Introduction

5G has already been deployed and commercialized on a large scale worldwide. However, with a new round of global technology industry upgrading, the total amount of data that needs to be transmitted is increasing, and new wireless communication technologies still need to be studied to adapt to new application scenarios and higher demands. As one of the most important technologies in physical wireless communication, channel coding is of great significance to improving system reliability^[1].

Spatially coupled low density parity check (SC-LDPC) codes have received widespread attention due to their threshold saturation^[2]. Threshold saturation is the belief propagation (BP) threshold of SC-LDPC codes that approaches the maximum a posteriori (MAP) threshold of their underlying block codes if coupling chain length $L \rightarrow \infty$ ^[3]. There are many results on the optimization of SC-LDPC codes. Ref. [4] introduces a procedure for constructing QC-SC-LDPC codes with a girth of at least eight, and the design leads to improved decod-

ing performance, particularly in the error floor compared with random constructions. Ref. [5] designs time-invariant spatially coupled low-density parity-check (SC-LDPC) codes with a small constraint length and low error floor. This is achieved by eliminating some of the dominant trapping sets (TSs) of the codes.

RC-SC-LDPC codes can adapt to the changing conditions of time-varying channels while allowing transceivers to employ only one encoder/decoder. There are two ways of constructing RC-SC-LDPC codes. One is puncturing^[6]. In a rate-compatible puncturing scheme, the transmitter punctures coded symbols, and as a result of having fewer transmitted code symbols, the code rate is increased. The other is the extension^[7] by selecting a code with a high rate, and constructing a lower code rate by adding check bits. The code with a high bit rate is nested in a lower code. In addition to using the extension and puncturing alone to achieve SC-LDPC code rate compatibility, combining the two methods to design code rate compatibility makes it easier to cover more code rates while ensuring the performance of each code rate.

The raptor-like structure is convenient to achieve rate compatibility, and it has better performance at a low code rate^[8-9]. RL-LDPC codes have been adopted in the technical specifica-

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We designed the coupled matrix sequentially from the parity-check matrix for high code rates to the parity-check matrix for low code rates. In the complete code-rate compatible parity-check matrix, the low code-rate parity-check matrix is obtained by extending the high code-rate parity-check matrix. Therefore, we only couple the extended matrices to construct the low code rate SC-LDPC codes. The coupling relationship in the coupling matrix corresponding to the low code rate includes the coupling relationship in the coupling matrix corresponding to the high code rate. A matrix of RL-RC-LDPC codes is shown in Fig. 4(a). For the matrix $H_k (k = 1, 2, \dots, n)$, H_1 is the highest rate sub-matrix, and H_2, \dots, H_n is the expansion matrix. In the process of edge coupling, we only select the edge for coupling in the matrix $H_k (k = 1, 2, \dots, n)$. We set H_{k_0} , H_{k_1} and H_k to satisfy the following form:

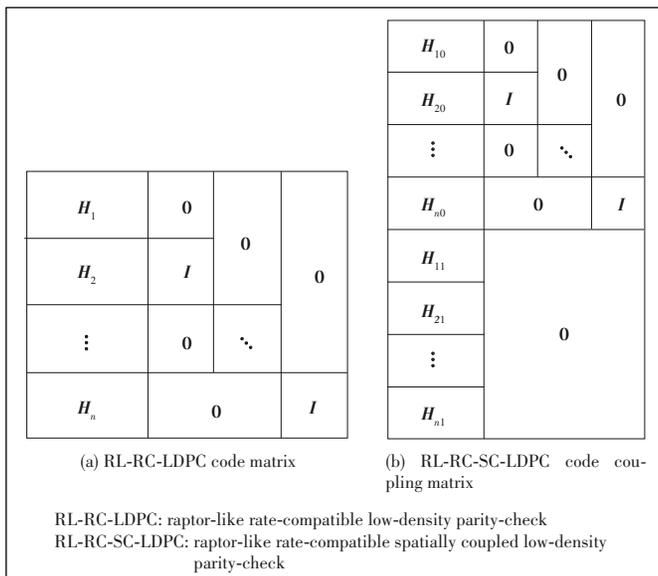
$$H_k = H_{k_0} + H_{k_1} (k = 1, 2, \dots, n). \tag{2}$$

Eq. (2) means all edges in H_{k_0} and H_{k_1} originate from H_k . This is set up to maximize performance through coupling without changing the degree distribution.

The complete RL-RC-SC-LDPC code coupling matrix is generated when the parity-check matrix for the lowest code rate is coupled as shown in Fig. 4(b).

Code-rate compatible coupling algorithms need to ensure a good coupling relationship at all code rates. Better coupling at each code rate means better performance benefits at each code rate. Thus, the selection of the side for coupling is more strict. The constraints on the edges selected for coupling are as follows:

- 1) Edges do not couple special structures in H_k ;
- 2) Avoid all zero columns and all zero rows in H_{k_0} ;
- 3) According to the check node degree distribution in H_k ,



▲ Figure 4. RL-RC-LDPC code matrix and constructed RL-RC-SC-LDPC code coupling matrix

avoid the edges of check nodes with relatively small coupling degrees.

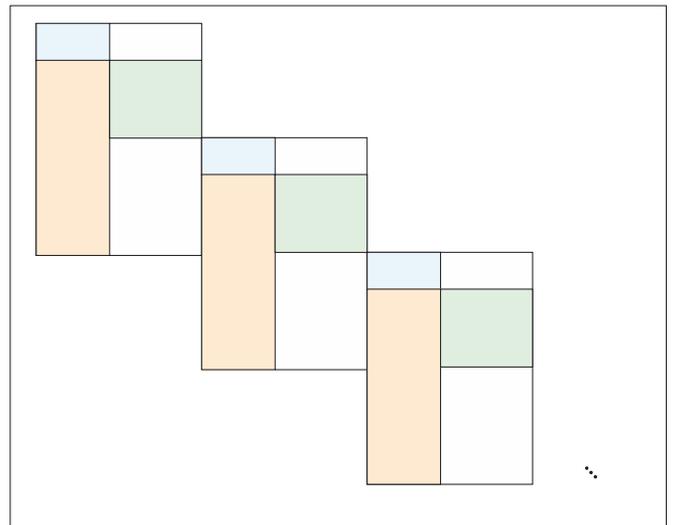
4) Under constraint 3), the edge selection is coupled row by row in H_k , and only one edge is selected in the selected row.

Next, the effect of constraint conditions in the rate-compatible coupling algorithm is explained. To reduce coding complexity, the Raptor-like structure is often used in combination with a double diagonal structure. The double diagonal structure is the key to encoding and in order to encode correctly, the integrity of the double diagonal structure needs to be ensured. It is worth noting that constraint 1) is only for the matrix with the highest rate, and there is no double diagonal structure in other extension matrices. Constraint 2) avoids all-zero columns and all-zero rows in H_{k_0} , since their appearance will greatly affect decoding performance. Constraint 3) is to ensure that the selected number of edges is moderate, which will not cause too much error propagation, and will not weaken the coupling relationship. The performance of SC-LDPC will deteriorate if the coupling relationship is weakened. The row-by-row edge selection in constraint 4) is to connect more variable nodes with the next check node in the SC-LDPC code coupling chain under the above constraints, and increase the coupling relationship in the coupling chain to ensure that useful information is transmitted in the coupling chain, thereby improving the performance of the SC-LDPC code.

Finally, the coupling matrix can be expanded by the method of the cyclic permutation matrix, and the expanded coupling matrix can be copied by L and placed according to the definition of SC-LDPC code. Then an SC-LDPC code with coupling chain length L can be generated, the matrix of which is shown in Fig. 5.

4 Sliding Window RCA Algorithm

The RCA algorithm is described in Ref. [11], Refs. [12 - 13]



▲ Figure 5. Raptor-like rate-compatible spatially coupled low-density parity-check (RL-RC-SC-LDPC) codes

prove that the deviation of RCA from the accurate density evolution threshold is less than 0.01 dB. The RCA algorithm can accurately calculate the threshold of a matrix containing a large number of variable nodes with degree 1. The SC-LDPC code can be considered as an LDPC code with a special structure from the overall check matrix, so the RCA algorithm can also be used to calculate the SC-LDPC code threshold. However, when using the RCA algorithm, due to the special coupling chain structure of the SC-LDPC code, the check matrix is relatively large, and its structural advantages cannot be highlighted. Based on the RCA algorithm, we proposed a sliding window RCA algorithm. The sliding window RCA algorithm can accurately calculate the SC-LDPC code threshold in the case of large L .

Set the signal-to-noise ratio to s_e . e is the edge connecting the variable node and the check node. q_e is the variable node passing information to the check node through edge e . r_e is the check node passing information to the variable node through edge e . E_v is a set of edges connected to variable node v . E_c is a set of edges connected to check node c . For a variable node of degree 1, its value will always be determined by the signal-to-noise ratio of the input channel, independent of the number of iterations. In this case, the overall reliability $Q_v^{(n)}$ of the variable node needs to replace the reliability of the edge. Let T be the stop threshold and f_{RCA} be a binary-valued function. When s_e is higher than the decoding threshold, the value is 1, otherwise, it is 0. Set the sub-matrix size of the constructed SC-LDPC code to $a \times b$. The size of the window is W , and the current sliding times is w . The maximum number of iterations is set to N .

The sliding window RCA algorithm is as follows:

Algorithm 1. sliding window RCA algorithm

1. **if** $w=1$
for edges e connected to punctured variable nodes in a sliding window, set $q_e^{(0)} = 0$. For all other edges, set $q_e^{(0)} = s_e$;
else
edges connecting the 1st to the b -th variable node in the window are initialized to $Q_v^{(n)}$ after the previous window converges. From $b + 1$ to Wb variable nodes in the window and edges connected to punctured variable nodes in the sliding window, set $q_e^{(0)} = 0$, for other edges, set $q_e^{(0)} = s_e$.
end
2. **for** the number of iterations n ,
compute (q_e^n, r_e^n) in the sliding window
$$r_e^{(n)} = \sum_{i \in E_c, e} R(q_i^{(n-1)})$$

$$q_e^{(n)} = q_e^{(0)} + \sum_{i \in E_v, e} R(r_i^{(n)})$$

3. Calculate $Q_v^{(n)}$ for each iteration in the sliding window
$$Q_v^{(n)} = Q_v^{(0)} + \sum_{e \in E_v} R(r_e^{(n)})$$

$$Q_v^{(0)} = \begin{cases} 0 & \text{if } v \text{ is punctured} \\ s_c & \text{otherwise} \end{cases}$$
 4. Set the minimum value of $Q^*(n)$ to $Q_v^{(n)}$. At each iteration,
if $w=1$,
Count the $Q^*(n)$ of 1 to b variable nodes in the sliding window.
else
Count the $Q^*(n)$ of $b + 1$ to $2b$ variable nodes in the sliding window.
end
 5. **if** $Q^*(n) > T$
 $f_{RCA} = 1$, slide to the next window until the last window;
else
 $f_{RCA} = 0$, the number of iterations is increased by 1 until the maximum number of iterations N is reached.
end
- end**

5 Simulation Results

5.1 Comparison with 5G NR LDPC Code Threshold

We use BG_1 in the 5G NR LDPC code as the base matrix to construct the SC-LDPC code and compare the threshold of the 5G NR LDPC code. The coupling chain length L of the constructed SC-LDPC code is 100, using RA termination^[14], corresponding to the base matrix rate 1/3, 2/5, 1/2, 2/3, 3/4, 5/6, and 8/9. The rate losses are 0.002 3, 0.002 5, 0.002 7, 0.002 6, 0.002 4, 0.002 1, and 0.001 8. When using the sliding window RCA algorithm, set the window size to 10, and the iteration number in each window is 250. The threshold results are shown in Table 1.

The coupling chain length L is 100, and a repeat-accumu-

▼ **Table 1.** Threshold results corresponding to the base matrix rate 1/3, 2/5, 1/2, 2/3, 3/4, 5/6, and 8/9

Rate	SC-LDPC Code Threshold/dB	5G NR LDPC Code Threshold/dB	Gap/dB
1/3	-0.338 6	-0.218 8	-0.119 8
2/5	-0.073 1	0.099 4	-0.172 5
1/2	0.353 9	0.450 3	-0.096 4
2/3	1.238 0	1.370 9	-0.132 9
3/4	1.681 4	1.797 0	-0.115 6
5/6	2.350 7	2.439 4	-0.088 7
8/9	3.073 9	3.145 2	-0.071 3

NR: new radio SC-LDPC: spatially coupled low density parity check

late (RA) method is used to terminate. When the window size is 10, the iteration number in each window is 250. The SC-LDPC code coupling matrix threshold is calculated with \mathbf{BG}_2 as the base matrix and the information bit K_b is 10, corresponding to the base matrix rate 1/5, 1/3, 2/5, 1/2, and 2/3. The rate losses are 0.001 7, 0.002 4, 0.002 7, 0.003, and 0.003 1. The thresholds compared with 5G NR LDPC codes are as follows.

The threshold results for each code rate in Tables 1 and 2 show that the thresholds for the RL-RC-SC-LDPC codes constructed in this paper are closer to the Shannon limit than those for the 5G NR LDPC codes. It is worth noting that this is the threshold value calculated when L is 100. If L continues to be increased, the threshold can approach the Shannon limit further.

5.2. Performance Compared with 5G NR LDPC Codes

In this section, we evaluate the performance of the proposed RL-RC-SC-LDPC codes over the additive white Gaussian noise (AWGN) channel using binary phase shift keying (BPSK) modulation. For decoding, we use sliding window decoding^[15]. When the window size is 2, BP decoding is used in the window, and the number of iterations is 30. 5G NR LDPC code decoding adopts BP decoding and BPSK modulation over the AWGN channel. The number of iterations is 60. We use the SC-LDPC code constructed by the base matrix of the 5G NR LDPC code, which ensures that the number of edges of the 5G NR LDPC code is the same as the number of edges of the constructed SC-LDPC code. We set the number of iterations for BP decoding of 5G NR LDPC codes to be the product of the number of iterations in the SC-LDPC code window and the size of the window. This ensures that SC-LDPC codes and 5G NR LDPC codes have the same decoding complexity. We constructed a sub-matrix with an information bit length of 512 bits, which compares performance with 5G NR LDPC codes under 8 rates. The coupling chain length L of the constructed SC-LDPC code is 100, using RA termination.

Fig. 6 shows the bit error ratio (BER) and frame error ratio (FER) simulation curves of the SC-LDPC code with a sub-matrix information bit length of 512 bits and code rates of 1/5, 1/3, 2/5, and 1/2. When the BER is 10^{-5} , it is at least about 0.5 dB better than the 5G NR LDPC code with the same information bit length and rate. When the FER is 10^{-4} , the 5G NR LDPC code with the same information bit length and the same code

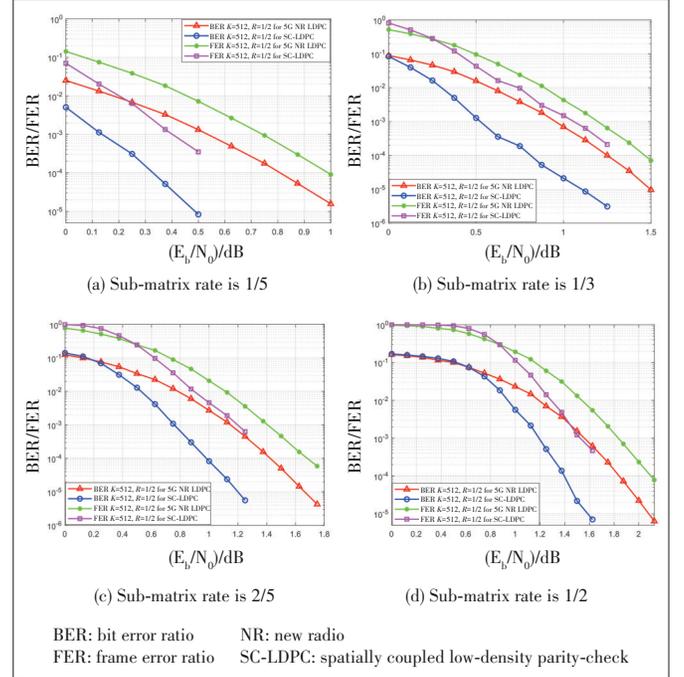
▼ Table 2. Threshold results corresponding to the base matrix rate 1/5, 1/3, 2/5, 1/2, and 2/3

Rate	SC-LDPC Code Threshold/dB	5G NR LDPC Code Threshold/dB	Gap/dB
1/5	-0.724 0	-0.690 3	-0.033 7
1/3	-0.321 1	-0.268 8	-0.052 3
2/5	-0.051 2	0.029 4	-0.080 6
1/2	0.376 3	0.470 3	-0.094 0
2/3	1.261 1	1.310 9	-0.049 8

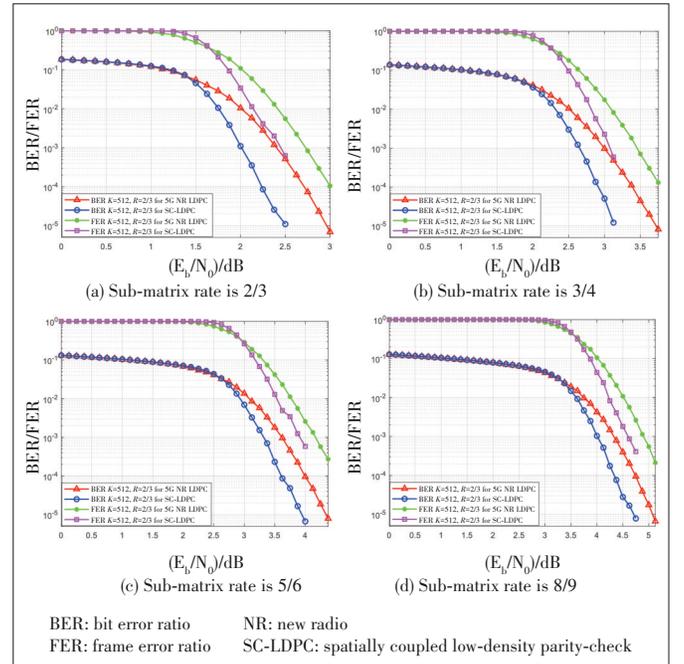
NR: new radio SC-LDPC: spatially coupled low density parity check

rate is at least 0.1 dB and at most 0.3 dB.

Fig. 7 shows the BER and FER simulation curves of the SC-LDPC code with a sub-matrix information bit length of 512 bits and code rates of 2/3, 3/4, 5/6 and 8/9. When the BER is 10^{-5} , the 5G NR LDPC code with the same information bit length and the same code rate is at least 0.1 dB and at most 0.4 dB. When the FER is 10^{-4} , the 5G NR LDPC code with the same information bit length and the same code rate is at least 0.1 dB and at most 0.3 dB.



▲ Figure 6. Performance comparison under 4 rates of 1/5, 1/3, 2/5, and 1/2



▲ Figure 7. Performance comparison under 4 rates of 2/3, 3/4, 5/6, and 8/9

The analysis above shows that the RL-RC-SC-LDPC code proposed in this paper has a significant performance gain compared with the 5G NR LDPC code, indicating that the rate-compatible coupling algorithm proposed in this paper has certain advantages.

6 Conclusions

In this paper, we propose a family of RL-RC-SC-LDPC codes from RL-RC-LDPC block codes. We have designed a rate-compatible coupling algorithm, which can improve performance under multiple code rates. We use the 5G NR LDPC code as the base matrix, and construct the RL-RC-SC-LDPC code through the coupling algorithm proposed in this paper. The simulation results show that the performance of the SC-LDPC code we designed can surpass the 5G NR LDPC code under the same parameters. To calculate the RL-RC-SC-LDPC code threshold with a large L , we improve the RCA algorithm and propose a sliding window RCA algorithm. The code family shows improved thresholds close to the Shannon limit to 5G NR LDPC codes for the AWGN channel.

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