

# A Survey on Low Complexity Detectors for OTFS Systems



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**Abstract:** The newly emerging orthogonal time frequency space (OTFS) modulation can obtain delay-Doppler diversity gain to significantly improve the system performance in high mobility wireless communication scenarios such as vehicle-to-everything (V2X), high-speed railway and unmanned aerial vehicles (UAV), by employing inverse symplectic finite Fourier transform (ISFFT) and symplectic finite Fourier transform (SFFT). However, OTFS modulation will dramatically increase system complexity, especially at the receiver side. Thus, designing low complexity OTFS receiver is a key issue for OTFS modulation to be adopted by new-generation wireless communication systems. In this paper, we review low complexity OTFS detectors and provide some insights on future researches. We firstly present the OTFS system model and basic principles, followed by an overview of OTFS detector structures, classifications and comparative discussion. We also survey the principles of OTFS detection algorithms. Furthermore, we discuss the design of hybrid OTFS and orthogonal frequency division multiplexing (OFDM) detectors in single user and multi-user multi-waveform communication systems. Finally, we address the main challenges in designing low complexity OTFS detectors and identify some future research directions.

**Keywords:** high mobility wireless communications; OTFS; ISFFT; SFFT; delay-Doppler diversity; iterative maximum ratio combining (MRC) detection; message passing detection

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## 1 Introduction

The new-generation mobile communication systems<sup>[1]</sup> are the key enabler for the digital society in the next ten years and are expected to satisfy the requirements for high mobility applications such as vehicle-to-everything (V2X) services<sup>[2-3]</sup>, high-speed railway services<sup>[4-5]</sup>, as

well as unmanned aerial vehicles (UAV), which require the support of high mobility up to 500 - 1 000 km/h with acceptable quality of service (QoS)<sup>[1,6]</sup>.

However, high mobility wireless communications suffer from high Doppler spread, and the transmitted signals experience time-frequency doubly selective channel<sup>[7]</sup>. High Doppler spread will result in very serious inter-carrier interference (ICI), especially in orthogonal frequency division multiplexing (OFDM) systems. Another challenge is to perform channel estimation to obtain exact channel state information (CSI) of fast time-variant channels, even to the extent that the reported CSI is outdated. These challenges will seriously reduce the performance of conventional OFDM systems. To tackle the challeng-

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es of high mobility, learning-based channel estimation, flexible subcarrier spacing and length of cyclic prefix (CP), double demodulation reference signals (DMRS), i. e., front-loaded DMRS and additional DMRS with configurable time-domain density, have been studied. However, these methods still treat high mobility as a negative factor, which results in very limited performance improvements of OFDM systems.

Recently, the orthogonal time frequency space (OTFS) modulation technology<sup>[8-9]</sup> has been proposed for high mobility wireless communications, and attracted increasing attention due to its excellent performance. This new two-dimensional (2D) modulation transforms high mobility into a positive factor by introducing inverse symplectic finite Fourier transform (ISFFT)-based pre-processing before OFDM modulation and symplectic finite Fourier transform (SFFT)-based post-processing after OFDM demodulation. With ISFFT/SFFT transforms, delay-Doppler (DD) domain is introduced in OTFS systems and the modulated symbols are transmitted in DD domain rather than time-frequency (TF) domain. The equivalent DD channel exhibits excellent features of separability, stability, compactness, and possible sparsity<sup>[9]</sup>, which enables OTFS systems to obtain delay-Doppler diversity gain. Additionally, these excellent features are also beneficial for performing channel estimation under high mobility environments. OTFS modulation has also been submitted to 3GPP as a candidate waveform for 5G systems<sup>[10-12]</sup>, and is regarded as a promising waveform for next-generation wireless communications<sup>[13]</sup>.

However, since each modulated symbol is spread to the whole TF resource grid by ISFFT operation in OTFS systems, the number of equivalent DD channel dimensions is larger than that of OFDM systems, which dramatically increases the complexity of signal detection. To address this challenge, some efforts have been devoted to the research of low complexity OTFS detector structures such as decision feedback equalizer (DFE)<sup>[14]</sup>, iterative maximum ratio combining (MRC) detector<sup>[15-16]</sup>, non-iterative joint TF- and DD-domain detector<sup>[17]</sup>, iterative joint time- and DD-domain detector<sup>[18]</sup>, non-iterative MRC detector with compensation<sup>[19]</sup>, learning-based detector<sup>[20-23]</sup>, and separate low complexity OTFS detector<sup>[24]</sup>. Several OTFS detection algorithms, including linear minimum mean square error (MMSE) and zero-forcing (ZF)<sup>[25-29]</sup>, message passing (MP)<sup>[30-35]</sup> and its variants like approximate message passing (AMP)<sup>[34-36]</sup>, MRC<sup>[15-16]</sup>, joint MP and MRC<sup>[37]</sup>, hybrid maximum a posteriori (MAP) and parallel interference cancellation (PIC)<sup>[38]</sup>, expectation propagation (EP)<sup>[39]</sup>, variational Bayes (VB)<sup>[40]</sup>, and iterative least squares minimum residual (LSMR)<sup>[41]</sup>, have been studied.

In this paper, a comprehensive survey on OTFS detector structures and detection algorithms is provided. We compare the advantages and disadvantages of each OTFS detector structure and detection algorithm, which can provide some insights for future research. We also provide classifications for OTFS detectors from different dimensions. Furthermore, we

study a hybrid OFDM-OTFS multi-waveform detection framework. Finally, we discuss some challenges for low complexity OTFS detectors, and identify some future research directions. The rest of the paper is organized as follows. A brief discussion on the OTFS system model and the principles of OTFS modulation are given in Section 2. In Section 3, a survey on the state-of-the-art OTFS detector structures is provided, while the research progress on OTFS detection algorithms is given in Section 4. In Section 5, a hybrid OTFS-OFDM multi-waveform detection framework is discussed briefly, while Section 6 discusses the research challenges and identifies some future research directions, followed with conclusions.

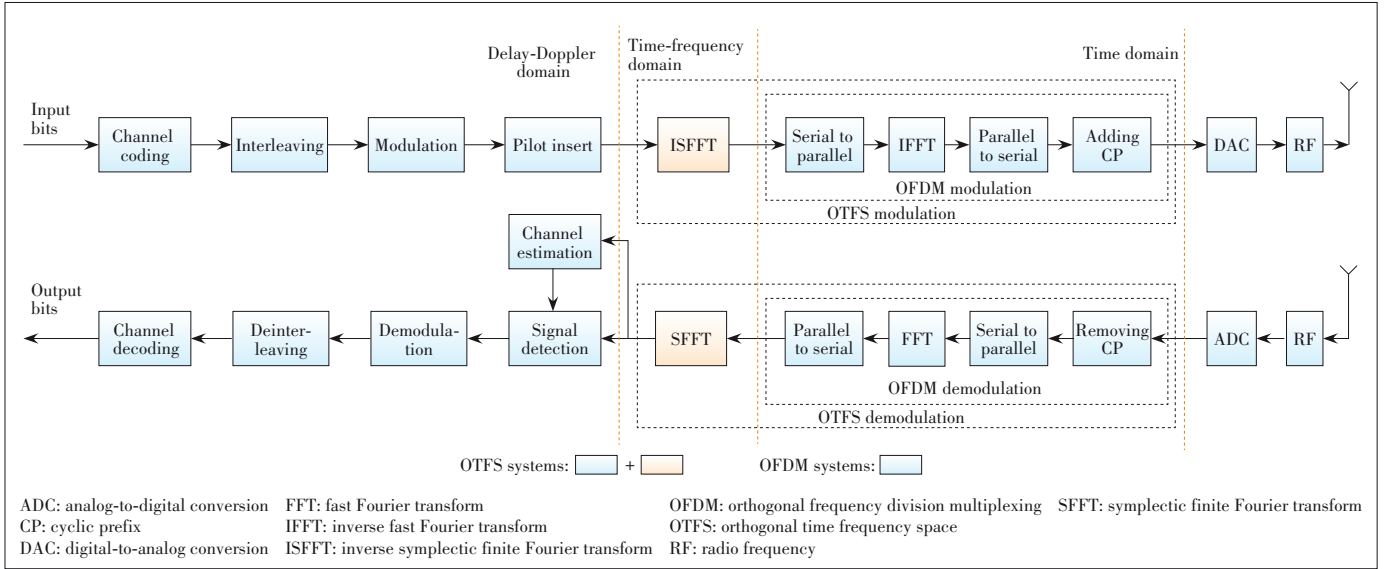
## 2 Basic Principles of OTFS Modulation

The OTFS system model is shown in Fig. 1, which includes OTFS transmitter and receiver structures. Compared with OFDM systems, OTFS systems add ISFFT-based transform precoding before OFDM modulation at the transmitter side, while SFFT-based post-processing is employed after OFDM demodulation at the receiver side. From the perspective of system structures, OTFS systems can be regarded as a type of precoded OFDM systems and can be easily compatible with OFDM systems. With the introduction of ISFFT/SFFT transform, a new domain, i. e., DD domain, is introduced. As a result, there are three domains in OTFS systems: DD domain, TF domain and time domain, while OFDM systems only have TF and time domains.

Considering an OTFS system with an  $N \times M$  DD resource grid, at the OTFS transmitter side, the modulated symbols and pilots are mapped to the DD resource elements. The signal carried by the  $(k, l)$ -th DD resource element is denoted by  $x^{DD}[k, l]$  for  $k = 0, 1, \dots, N-1, l = 0, 1, \dots, M-1$ . Then, the symbols  $x^{DD}[k, l]$  in the DD domain are converted to the symbols  $x^{TF}[n, m]$  in the TF domain using the ISFFT as

$$x^{TF}[n, m] = ISFFT(x^{DD}[k, l]) = \frac{1}{\sqrt{MN}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x^{DD}[k, l] e^{j2\pi(\frac{nk}{N} - \frac{ml}{M})}$$

for  $n = 0, 1, \dots, N-1, m = 0, 1, \dots, M-1$ . Next, the signals  $x^{TF}[n, m]$  in the TF domain is converted to the symbols in the time domain signal as  $x(t) = IFFT(x^{TF}[n, m]) = \frac{1}{\sqrt{MN}} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} x^{TF}[n, m] g_{tx}(t - nT) e^{j2\pi m \Delta f (t - nT)}$  and is transmitted through the channel. At the OTFS receiver side, the received signal in the time domain is  $y(t) = \int_{\nu} \int_{\tau} h(\tau, \nu) x(t - \tau) e^{j2\pi \nu (t - \tau)} d\tau d\nu$ . After OFDM demodulation (i. e., FFT transform), the symbols in the TF domain are denoted by  $y^{TF}[n, m]$ . Then, applying SFFT on  $y^{TF}[n, m]$ , the symbols in the DD domain can be obtained as  $y^{DD}[k, l] = SFFT(y^{TF}[n, m]) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} y^{TF}[n, m] e^{-j2\pi(\frac{nk}{N} - \frac{ml}{M})}$ . Finally, the transmitted symbols  $x^{DD}[k, l]$  can be recovered from  $y^{DD}[k, l]$  through the OTFS detector.

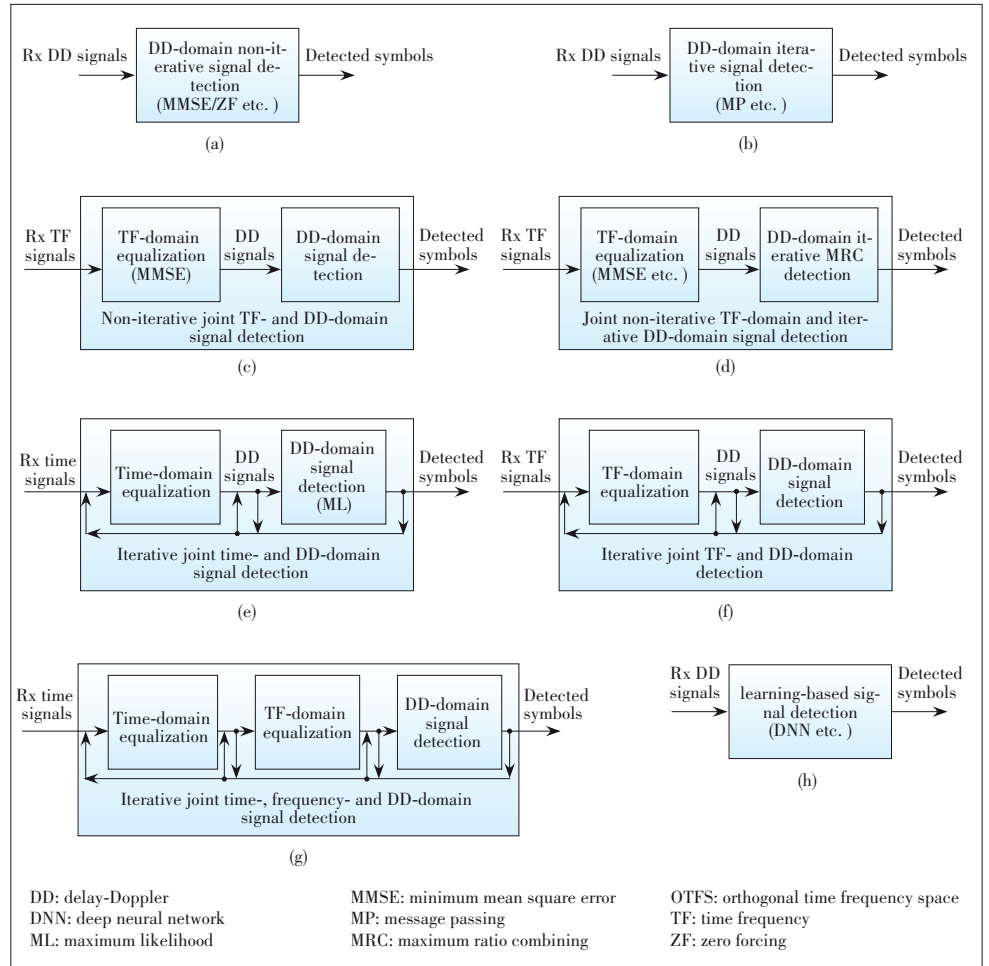


▲ Figure 1. OTFS system model

As shown in the expression for ISFFT, a DD symbol carried by a DD resource element is spread to all the TF resource elements, which enables OTFS systems to obtain full diversity. Further, since the pilots inserted in DD domain are also spread to all the TF resource elements, the equivalent DD channels obtained by channel estimation have the average channel gain. Due to delay and Doppler spread, the received symbols in DD domain are interfered with the neighboring symbols. Therefore, the main challenges for OTFS systems focus on the OTFS receiver, which needs to design very low complexity detectors, while the OTFS transmitter is relatively simple, as it only needs to add ISFFT operation before OFDM modulation.

### 3 OTFS Signal Detector Structures

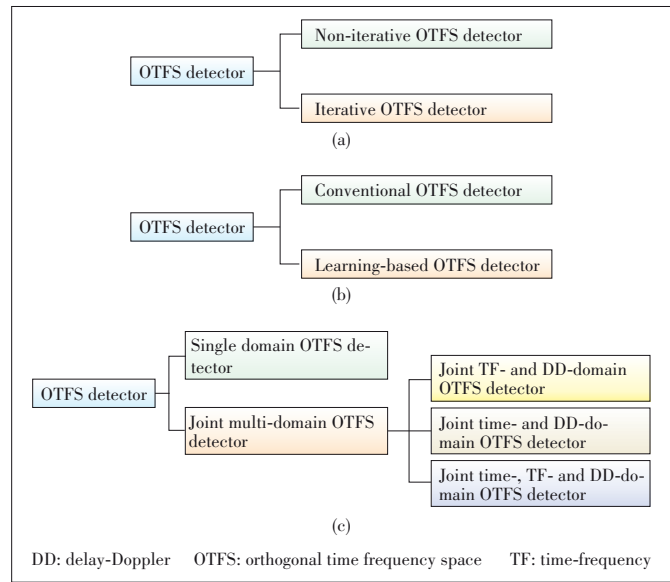
Several works have been devoted to studying low complexity OTFS detectors. Fig. 2 illustrates several popular OTFS detector structures, including DD-domain non-iterative detector<sup>[25-29]</sup>, DD-



▲ Figure 2. OTFS detector structures: (a) DD-domain non-iterative OTFS detector; (b) DD-domain iterative OTFS detector; (c) non-iterative joint TF- and DD-domain OTFS detector; (d) joint non-iterative TF-domain and iterative DD-domain OTFS detector; (e) iterative joint time- and DD-domain OTFS detector; (f) iterative joint TF- and DD-main OTFS detector; (g) iterative joint time-, TF- and DD-main OTFS detector; (h) learning-enabled OTFS detector

domain iterative detector<sup>[30-35]</sup>, non-iterative joint TF- and DD-domain detector<sup>[17]</sup>, joint non-iterative TF-domain and iterative DD-domain detector<sup>[15-16]</sup>, iterative joint Time- and DD-domain detector<sup>[18]</sup>, iterative joint TF- and DD-main detector, and learning-enabled detector<sup>[20-23]</sup>. According to the number of domains involved in detection processing, these OTFS detectors can be divided into two categories: the single-domain OTFS detector and joint multi-domain OTFS detector. With the need of iteration, these OTFS detectors can be divided into the non-iterative OTFS detector and iterative OTFS detector. These OTFS detectors can also be divided into the conventional OTFS detector and learning-based OTFS detector. The detailed classifications of OTFS detectors are shown in Fig. 3. A summary of OTFS detectors is illustrated in Table 1.

The DD-domain non-iterative OTFS detector shown in Fig. 2(a) achieves signal detection in the DD domain by using non-iterative detection algorithms like MMSE/ZF, spherical detection, maximum likelihood (ML) detection, etc., where MMSE/ZF is popular and has also been adopted by 4G/5G systems due to its low complexity, while spherical detection and ML



▲ Figure 3. OTFS detector classifications: (a) non-iterative and iterative OTFS detectors; (b) single domain and multi-domain OTFS detectors; (c) conventional and learning-based OTFS detectors

▼ Table 1. Summary of OTFS detector structures

Ref.	Detector Structure	Detector Structure Type	Domain	Basic Idea	Advantage	Disadvantage
Refs. [25] - [29]	Single-domain OTFS detector	DD-domain non-iterative OTFS detection	DD domain	Adopting non-iterative detection algorithms (e.g., MMSE/ZF) in DD domain	Signal detection is only performed in DD domain; Non-iterative signal detection algorithms are relatively low complexity.	Non-iterative signal detection algorithms suffer from some performance loss.
Refs. [30] - [35]	OTFS detector	DD-domain iterative OTFS detection	DD domain	Adopting iterative detection algorithms, like MP/AMP and MRC etc., in DD domain	Iterative detection algorithms can achieve better performance.	Iterative detection will increase the complexity of algorithm design; the convergence of algorithms should be analyzed and ensured.
Ref. [17]		Non-iterative joint TF- and DD-domain OTFS detection	TF domain and DD domain	Joint TF- and DD- domain processing with non-iterative detection algorithms	Joint multi-dimension processing can achieve better detection performance; Joint multi-dimension processing can relax the processing requirements in DD domain.	Joint multi-dimension processing increases the complexity of designing OTFS detector.
Refs. [15] and [16]	Joint multi-domain OTFS detector	Joint non-iterative TF-domain and iterative DD-domain OTFS detection	TF domain and DD domain	Employing TF MMSE equalizer to provide good initials for DD-domain iterative MRC detector	Introducing non-iterative TF MMSE equalizer can accelerate the convergence of DD-domain iterative MRC detector; iterative MRC detector can fully merge separable taps to obtain better performance.	TF detection is needed to provide initial estimates; iteration processing increases the complexity; need to add null symbols to construct full channel matrix.
Ref. [18]		Iterative joint time- and DD-domain OTFS detection	Time domain and DD domain	Joint processing of time and DD domains to form a large iterative detection loop.	Iterative joint time- and DD-domain detection can achieve better performance and faster convergence by fully utilizing time- and DD-domain information.	Iterative joint time- and DD-domain detection increases the complexity of designing OTFS detector; a large amount external information exchange is inevitable.
Ref. [18]		Iterative joint TF- and DD-main OTFS detection	TF domain and DD domain	Joint TF and DD domains that form a large iterative detection loop.	Iterative joint TF- and DD-domain detection can achieve better performance and faster convergence by fully utilizing TF- and DD-domain information.	Iterative joint TF- and DD-domain detection increases the complexity of designing OTFS detector; a large amount external information exchange is inevitable.
Refs. [20] - [23]		Learning-based OTFS detection	DD domain	Using machine learning techniques to perform signal detection in DD domain or estimate some parameters in conventional OTFS detector.	It is relatively simple to design learning-based signal detection as a black box without understanding expert knowledge of OTFS detection; better detection performance is achieved.	Learning-based detection is un-explainable; more computing capability is required; massive training and testing datasets are necessary.

AMP: approximate message passing DD: delay-Doppler MMSE: minimum mean square error MP: message passing MRC: maximum ratio combining  
 OTFS: orthogonal time frequency space TF: time-frequency ZF: zero forcing

detection are very complex. In general, the DD-domain non-iterative signal detector adopts MMSE/ZF algorithms. Without iteration operation, the computational complexity and processing delay of MMSE/ZF are small, but at the cost of detection performance loss.

The DD-domain iterative OTFS detector shown in Fig. 2(b) also achieves signal detection in the DD domain, but uses iterative detection algorithms like MP and its improved algorithms, and the EP algorithm<sup>[39]</sup>. These algorithms iteratively update information to achieve better detection performance. However, the iteration operation brings some extra computational complexity. Additionally, the convergence of iterative detection algorithms needs to be considered. In Ref. [39], the iterative EP algorithm and its improvement named Approximate EP (AEP) were studied. They exhibit better bit error rate (BER) performance than MMSE, MP, MRC rank and VB algorithms.

The non-iterative joint TF- and DD-domain OTFS detector shown in Fig. 2(c) can be considered as an improvement of the DD-domain non-iterative OTFS detector, which utilizes both TF- and DD-domain information to improve the detection performance. In Ref. [17], a sliding window-assisted MMSE (SW-MMSE) equalization in the TF domain was studied, and a DD equalizer like decision feedback equalizer (DFE) was introduced. The computation complexity of this non-iterative two-stage equalizer is lower than conventional MMSE, and the BER performance is also better than conventional MMSE.

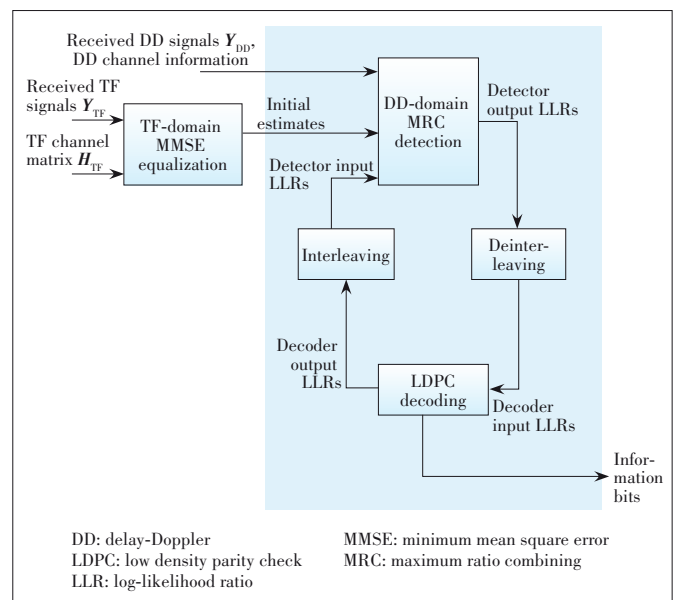
The joint non-iterative TF-domain and iterative DD-domain OTFS detector shown in Fig. 2(d) can be regarded as an improvement of the DD-domain iterative OTFS detector, in which the non-iterative TF-domain equalizer provides good initials for the iterative DD-domain OTFS detector to improve its convergence performance. In Refs. [15] and [16], an iterative MRC detector with initial estimates from the output of TF-domain MMSE equalizer was studied, as shown in Fig. 4. The results show that the iterative MRC detector with initial estimates can achieve better BER performance than that without TF-domain MMSE equalization, iterative MPA or MMSE. Considering spatial correlation at the receiver antennas, a sample-based method to estimate such correlation and the optimized combining weights for MRC from the estimated correlation matrix were studied in Ref. [42].

The iterative joint time- and DD-domain OTFS detector shown in Fig. 2(e) forms a large iteration loop among the time domain and DD domain, which is expected to obtain better performance and lower computational complexity by exploiting time domain channel sparsity and DD domain symbol constellation constraints. In Ref. [18], the iterative joint time- and DD-domain signal detector was studied, which adopted an L-MMSE estimator in the time domain and a symbol-by-symbol detection in the DD domain. The results show that this iterative joint time- and DD-domain signal detector could achieve almost the same error performance as the maximum-likelihood

sequence detection even in the presence of fractional Doppler shifts, and the computational complexity associated with the domain transformation was low.

The iterative joint TF- and DD-main OTFS detector shown in Fig. 2(f) can be regarded as an improvement of non-iterative joint TF- and DD-main signal detector. Similar as the iterative joint time- and DD-domain OTFS detector shown in Fig. 2(e), the iterative joint TF- and DD-main signal detector forms a large iteration loop among the TF domain and DD domain, which is expected to obtain better performance and faster convergence by utilizing TF- and DD-domain information. Furthermore, based on the OTFS detector shown in Fig. 2(f), an iterative joint time-, TF- and DD-main OTFS detector with time-domain equalization is shown in Fig. 2(g).

The learning-enabled OTFS detector shown in Fig. 2(h) uses advanced machine learning method to improve detection performance. In Ref. [20], to reduce the complexity of conventional MP detector in OTFS systems, a damped generalized approximate message passing (GAMP) algorithm was studied and deep learning (DL) was introduced to optimize damping factors. Its BER performance can outperform the classical GAMP algorithm and MP algorithm. In Ref. [21], a two-dimensional convolutional neural network (2D-CNN) based detector was studied to replace the conventional OTFS detector, and an MP-based data augmentation (DA) tool was employed to enlarge the training features of the input dataset and mitigate the effect of the channel variations to some degree, leading to improvement of the robustness and learning ability of the deep neural network (DNN). This 2D-CNN based detector can achieve superior performance compared with the MP detector and similar performance as the MAP detector with a very low complexity. In Ref. [22], a DD-domain symbol-level DDN detector was studied, which could achieve similar BER perfor-



▲ Figure 4. Iterative MRC detector<sup>[15]</sup>



mance as the full DDN detector and ML detector in static multipath channel with Gaussian noise, while it achieved better BER performance than the full DDN detector and ML detector in static multipath channel with non-Gaussian noise. In Ref. [23], a reservoir computing (RC)-based OTFS detector was studied, in which one-shot online learning was sufficiently flexible to cope with channel variations among different OTFS frames and explicit CSI was not required.

## 4 OTFS Detection Algorithms

OTFS detection algorithms include linear MMSE/ZF, MP and its improvements, MRC, MAP, EP, and VB algorithms. A summary of these detection algorithms including their computational complexity and BER performance is presented in Table 2.

### 4.1 Linear MMSE/ZF Detection Algorithm

Linear signal detection mainly includes MMSE and ZF, while MMSE has been adopted by 4G/5G OFDM systems, due to its low complexity. The detection matrices of classical MMSE and ZF in OTFS systems are  $\mathbf{G}_{MMSE} = (\mathbf{H}^H \mathbf{H} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}^H$  and  $\mathbf{G}_{ZF} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H$ , respectively. However, when these classical MMSE and ZF detection algorithms are used for OTFS systems directly, they suffer from very high complexity  $O(M^3 N^3)$ . This is because the number of dimensions of equivalent DD channel matrix is  $MN \times MN$  in OTFS

systems, which results in  $MN \times MN$  matrix inversion. To reduce the complexity of linear signal detection in OTFS systems, considering the sparsity and the block circulant nature of equivalent DD channel, some low complexity linear signal detection schemes have been studied.

In Ref. [27], the eigenvalues of  $\mathbf{G}_{MMSE}$  was computed from the eigenvalues of DD channel matrix  $\mathbf{H}$ , which can significantly reduce the complexity. This MMSE with low complexity is summarized as follows:

- 1) Compute the eigenvalues of each block of  $\mathbf{H}$ , by computing DFTs of the first row of each circulant block;
- 2) Compute the eigenvalues of  $\mathbf{H}$ ;
- 3) Compute the eigenvalues of  $\mathbf{G}_{MMSE}$ , by using the eigenvalues of  $\mathbf{H}$ ;
- 4) Compute  $\mathbf{G}_{MMSE} \mathbf{y}$ .

This idea was also adopted by Ref. [28] to study the detection in MIMO-OTFS systems. Unlike the SISO-OTFS channel, the eigenvalue matrix  $\mathbf{D}$  in MIMO-OTFS channel is not diagonal, however, the inverse of the  $\mathbf{D}_A$  constructed by the matrix  $\mathbf{D}$  can be performed block-wise by two steps: matrix partitioning and backtracking<sup>[28]</sup>.

The computational complexity of MMSE is mainly caused by large matrix inversion. Considering the sparsity of equivalent DD channel matrix and quasi-banded structure of matrices in MMSE detection, a lower-upper (LU) factorization-based low complexity MMSE detection algorithm was studied for OTFS systems with reduced CP<sup>[25]</sup> and full CP<sup>[29]</sup>, in which high complexity channel inversion is replaced by low complexity LU factorization operation. Further, the final estimate symbols step can be performed by Fast Fourier Transform (FFT). Its detailed procedure is illustrated in Fig. 5.

There are some other low complexity MMSE/ZF detection algorithms. For example, the one-tap MMSE detection algorithm studied in Ref. [26] achieved low complexity detection in pulse-shaped OTFS systems over doubly-dispersive channels, which only estimated the channel main diagonal and the self-interference power instead of interference cancellation and considered the power of the channel estimation error and self-interference as additional tuning variance parameters.

### 4.2 MRC Detection Algorithm

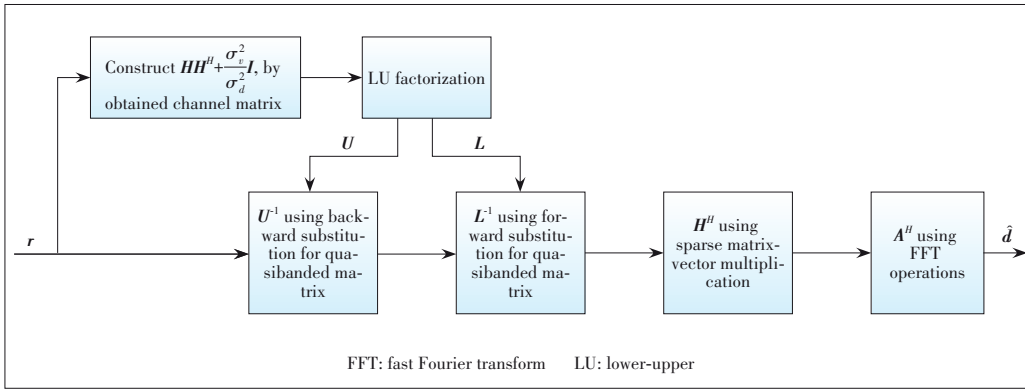
The MRC detection algorithm extracts received multipath components of the transmitted symbols in the delay-Doppler grid and combines them by using MRC to improve the signal-to-noise ratio (SNR) of the combined signal. The detailed steps of MRC algorithm are shown as follows<sup>[15-16]</sup>:

- 1) Construct circulant matrix having element  $\mathbf{K}_{m,l}$ , according to the channel Doppler spread vector at each delay tap;
- 2) Construct matrix  $\mathbf{R}$  by using a circulant matrix, where  $\mathbf{R}_m = \sum_{l \in \{l_i\}} \mathbf{K}_{m+l,l}^H \mathbf{K}_{m+l,l}$ ,  $m = 0, 1, \dots, M - l_{\max}$ ;
- 3) Construct the equations for the symbol vector estimates  $\mathbf{b}_m^l$ , by using the estimates of symbol vectors from previous iteration;

▼ Table 2. Summary of computational complexity and performance

Reference	Detection Algorithm	Algorithm Characteristic	Computational Complexity	Performance
Ref. [39]	Classical MMSE	Non-iterative	$O(M^3 N^3)$	
Ref. [27]	Low complexity MMSE	Non-iterative	$O(MN \log(MN))$	
Ref. [25]	lower-upper factorization-based MMSE	Non-iterative	$O(MN \log(N))$	
Refs. [31] and [32]	MP	Iterative	$O(2^Q IMNS)$	UAMP>EP>AEP
Ref. [33]	MF-MP-PC	Iterative	$O(IMN(2^{Q/2} + S))$	>MRC-rake
Ref. [34]	GAMP	Iterative	$O(2^Q IMNS)$	>VB
Ref. [35]	UAMP	Iterative	$O(IMN \log(MN)) + O(2^Q IMN)$	>MP
Ref. [36]	ICMP	Iterative	$O(2^Q IMNGS)$	>Classical MMSE
Refs. [15] and [16]	MRC-rake	Iterative	$O(IMN(L + \log(N))) + O(MN(L + \log(M)))$	$\geq$ low complexity MMSE
Ref. [39]	EP	Iterative	$O(IMN(2^Q + S))$	
	AEP	Iterative	$O(IMN(2^Q + S))$	
Ref. [40]	VB	Iterative	$O(2^Q IMNS)$	

AEP: approximate expectation propagation  
 EP: expectation propagation  
 GAMP: generalized approximate message passing  
 ICMP: iterative combining message passing  
 MF: matched filtering  
 MMSE: minimum mean square error  
 MP: message passing  
 MRC: maximum ratio combining  
 PC: probability clipping  
 UAMP: unitary approximate message passing  
 VB: variational Bayes



▲ Figure 5. LU factorization-based low complexity minimum-mean-square-error (MMSE) detection<sup>[25]</sup>

- 4) Construct  $\mathbf{g}_m$  according to  $\mathbf{K}_{m+L,L}$  and  $\mathbf{b}_m^l$ ;
- 5) Perform MRC of the estimates and obtain the output of the maximal ratio combiner,  $\mathbf{c}_m = \mathbf{R}_m^{-1} \cdot \mathbf{g}_m$ ;
- 6) Estimate all information symbol vectors by ML criterion;
- 7) Stop criteria by stopping iteration, when some conditions are satisfying, e.g., the number of iterations is up to the maximum number of iterations.

### 4.3 MP Detection Algorithm and Its Improvements

The MP algorithm<sup>[30-32]</sup> uses graphical models to decompose a hard problem into several easy sub-problems and iteratively solve them by passing messages between different types of nodes. The detailed processing steps of an MP algorithm are shown as follows<sup>[32]</sup>:

- 1) Message passings from observation nodes to variable nodes: Observation nodes compute the mean and variances of Gaussian random variables and pass them to variables nodes;
- 2) Message passings from variable nodes to observation nodes: Variable nodes update the probability mass function (PMF) of the alphabet and pass them to observation;
- 3) Convergence indicator: The convergence indicator is computed;
- 4) Update decision: The decision on the transmitted symbols is updated, if needed;
- 5) Stopping criteria: The iteration is stopped when some conditions are satisfying. Note that different stopping criteria will affect convergence and the number of iterations.

Popular MPA detectors still suffer from high complexity and high storage requirements, as well as error floor of BER performance at high SNRs. To further improve the MPA detector, a matched-filtering based message passing detector with probability clipping (MF-PC-MPD) for OTFS systems was studied in Ref. [33]. MF-PC-MPD first performs matched filtering on received OTFS signals, and then uses probability clipping to redistribute the probability if the probability distribution satisfies a certain condition, which makes the symbol variance fluctuate within a certain range and close to each other; in this way, the Gaussianity is retained.

In Ref. [34], a Gaussian approximate message passing (GA-

MP) detection was studied, which aimed at overcoming the performance degradation of MP detectors caused by non-ideal Gaussian interference due to the limited number of interfering symbols for a certain symbol. The GA-MP detector modeled the individual transmit signals by Gaussian distributions, rather than approximating the ISI. This detector outperforms the classical

MP detector by at least 1.5 dB at a BER of  $10^{-4}$ , with the same complexity order.

To overcome the performance loss of MP detector in the case of rich scattering environments or fractional Doppler shifts, a unitary approximate message passing (UAMP) detector was studied in Ref. [35]. Considering the equivalent DD channel is a block circulant matrix with circulant blocks which can be diagonalized using a 2D DFT matrix, the UAMP detector performs unitary transform by using a unitary matrix after receiving DD signals. As a result, the UAMP detector allows more efficient implementation with the FFT algorithm, and can achieve better BER performance than VB, MRC, MP, and AMP algorithms.

In Ref. [36], fractionally spaced sampling (FSS) was introduced to the OTFS receiver, which can be equivalent to a SIMO system, and then iterative combining message passing (ICMP) and turbo message passing (TMP) detectors were studied, by exploiting the sparsity of DD channel and the channel diversity gain via FSS. The ICMP detector combines two receiving channels and then performs message passing iteratively with the Gaussian approximation of the interference components. Considering there are two receiving channels in the FSS receiver, the TMP detector uses two individual MP equalizers with extrinsic log-likelihood ratios (LLRs) exchanging to form a turbo receiver.

The MP detection algorithm is based on the factor graph between variable nodes and observation nodes, and is very efficient for the sparse channel. However, its complexity will be increased when there are a large number of paths such as multi-antenna transmission. To overcome this challenge for OTFS systems with multi-antennas, a joint MP and MRC detection algorithm was studied in Ref. [37], which separated the Doppler frequency offsets (DFOs) in the spatial domain with a beamforming network to ensure the equivalent sparsity and obtained the best diversity by employing MRC to combine all beamforming branches. The main steps in each iteration of the joint MP-MRC algorithm are: 1) Each observation node passes the mean and variance of the interference terms to the connected variable nodes; 2) Each variable node updates the

PMF of alphabet symbols and then passes it back to the connected observation nodes; 3) The joint convergence indicator of all beamforming branches is calculated in the MRC fashion after each iteration. Finally, when the convergence is satisfying, the soft output of each transmitted symbol is computed, followed with hard decision.

#### 4.4 MAP Detection Algorithm

The MAP detection algorithm uses all received signals to estimate all transmitted symbols, which can be formulated as  $\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{A}^{NM \times 1}} \Pr(\mathbf{x}|\mathbf{y}, \mathbf{H})$ . Obviously, its complexity increases with exponent in  $NM$ <sup>[32]</sup>. To reduce the complexity, a near-optimal symbol-wise MAP detection algorithm was studied in Ref. [38], and its detection rule is expressed as  $x(k, l) = \arg \max_{x(k, l) \in \mathcal{A}} \Pr(x(k, l)|\mathbf{y}, \mathbf{H})$ .

#### 4.5 EP Detection Algorithm

The OTFS system can be represented by a sparsely-connected factor graph where each variable node (VN) is connected to factor nodes  $D$ . The main idea of EP algorithm<sup>[39]</sup> is to use a Gaussian distribution through distribution projection to approximate the sophisticated posterior distribution in the message updating steps, which leads to the complicated belief computation being replaced by means and variances computation. The detailed steps of EP algorithm are represented as follows<sup>[39]</sup>:

- 1) Compute the joint distribution  $p(\mathbf{x}_{DD}, \mathbf{y}_{DD})$ ;
- 2) Compute the likelihood function  $p(\mathbf{y}_b|\mathbf{x}_{DD})$ ;
- 3) Compute the means and variances passed from FNs and VNs as  $u_{f_b \rightarrow x_a}^i$  and  $v_{f_b \rightarrow x_a}^i$ ;
- 4) Compute the means and variances passed from VNs and FNs as  $u_{x_a \rightarrow f_b}^i$  and  $v_{x_a \rightarrow f_b}^i$ ;
- 5) Compute the a posteriori LLR of each coded bit as  $c_b^q$ ;
- 6) Stop criteria by stopping iteration, when some conditions are satisfied.

Note that the main computational complexity of the EP algorithm depends on the number of non-zero elements  $D$  of channel matrix. In case of rich scattering scenarios and fractional Doppler shift,  $D$  is relatively large. To further reduce the computational complexity, small channel coefficients can be approximated to a fixed value (e.g., the median value of these small elements) during the message passing from FNs to VNs, which is named channel coefficients-aware approximate EP (AEP) algorithm<sup>[39]</sup>.

#### 4.6 VB Detection Algorithm

The optimal MAP detection algorithm suffers from very high complexity, which increases exponentially with the size of data symbol vector. To reduce the complexity of MAP algorithm, a variational Bayes algorithm was studied in Ref. [40]. The main idea of VB algorithm is to find a distribution  $q(\mathbf{d})$  from a tractable distribution family as an optimized approxima-

tion of the a posteriori distribution  $p(\mathbf{d}|\mathbf{y})$ . The detailed procedures are shown as follows:

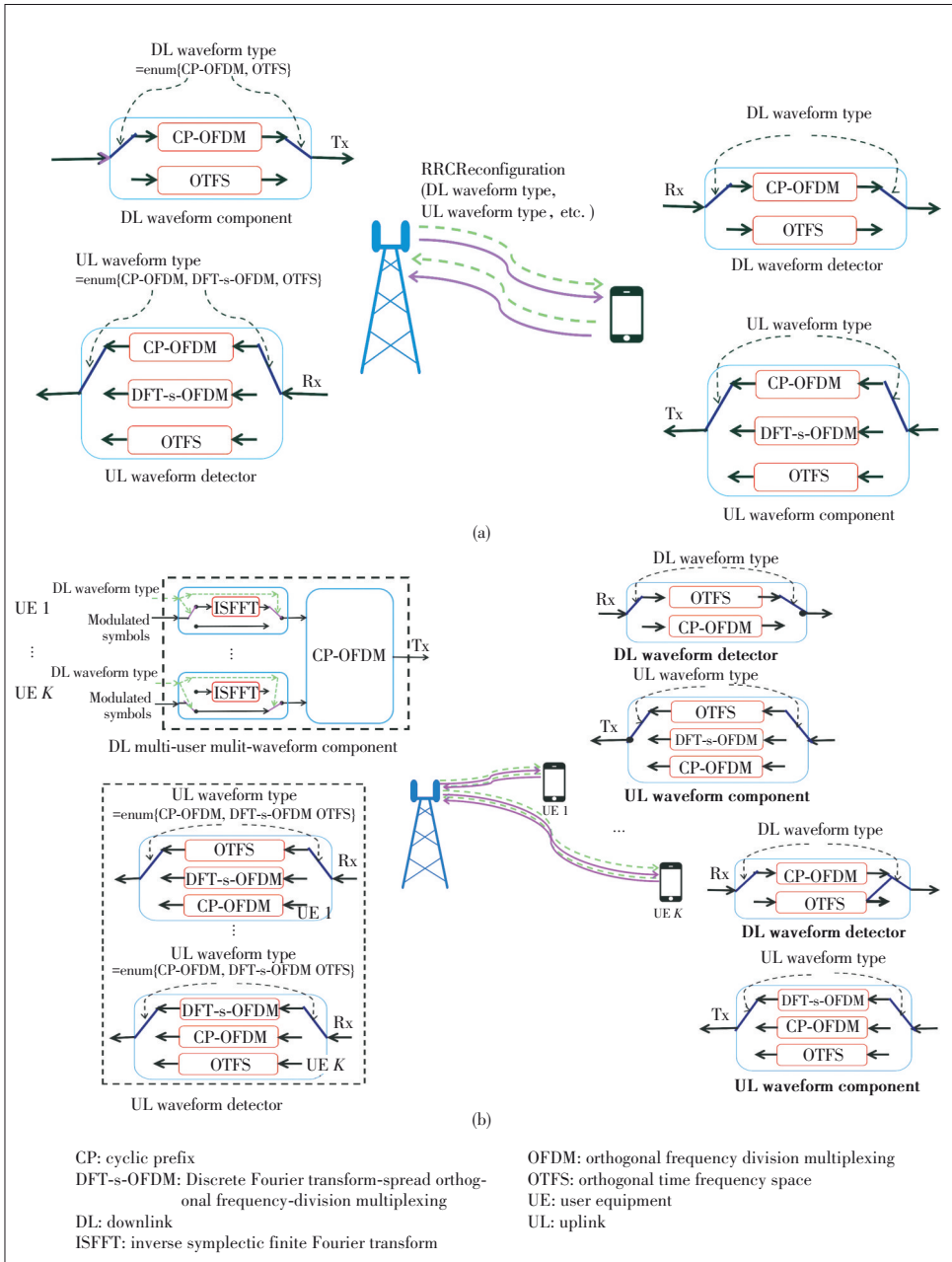
- 1) Formulate the approximation  $q^*(\mathbf{d})$  as an optimal problem by minimizing the Kullback-Leibler divergence;
- 2) Construct the approximation  $q(\mathbf{d})$  by mean field approximation as  $q(\mathbf{d}) = \prod_{k,l} q_{k,l}(d_{k,l})$ . Note that in this form, all variables are mutually independent;
- 3) Transform  $p(\mathbf{d}|\mathbf{y})$  into a pairwise form;
- 4) Obtain the variational function in the optimization problem in Step 1, by substituting  $q(\mathbf{d})$  and  $p(\mathbf{d}|\mathbf{y})$  into the optimization problem;
- 5) Find a stationary point of the variational function, by iteratively updating each local function  $q_{k,l}(d_{k,l})$ ;
- 6) Approximate a posteriori distributions for all the data symbols iteratively, resulting in the approximate marginals  $q_{k,l}^*(d_{k,l})$ ;
- 7) Estimate the transmitted symbols by finding the maximum of marginal distribution  $q_{k,l}^*(d_{k,l})$ .

## 5 Hybrid OFDM-OTFS Multi-Waveform Detector Structure

To satisfy the requirements for various scenarios and applications, mobile communication systems have evolved from single waveform to multi-waveform systems. For example, in 4G systems, high-spectrum efficiency CP-OFDM is adopted by the downlink, while the uplink adopts single-carrier frequency division multiple access (SC-FDMA) with a low peak to average power ratio (PAPR). In 5G systems, the downlink adopts CP-OFDM, while CP-OFDM and discrete Fourier transform-spread orthogonal frequency-division multiplexing (DFT-s-OFDM) with low-PAPR are adopted by the uplink. In general, when UE is in the cell center, UE can still obtain the expected QoS with low transmit power, thus UE can adopt CP-OFDM waveform with higher spectrum efficiency. When UE is at the cell edge, UE should increase transmit power to obtain the expected QoS, which requires UE to adopt DFT-s-OFDM waveform with low PAPR. Since OTFS exhibits excellent performance in high mobility environments, if OTFS is accepted by future mobile communication systems (FMCS), its downlink waveform will be CP-OFDM or OTFS, while its uplink waveform will be CP-OFDM, DFT-s-OFDM or OTFS. To determine each user's uplink (UL)/downlink (DL) waveform, the base station shall call UL and DL waveform decision algorithms with the input of user mobility speed and user type. Further, the base station also needs to dynamically switch user's downlink and uplink waveform type if some conditions are triggered.

Fig. 6 shows the hybrid OFDM-OTFS multi-waveform detector structure, in which Fig. 6(a) is for single user OTFS systems and Fig. 6(b) is for multi-user OTFS systems. The base station first determines the DL and UL waveform types accord-





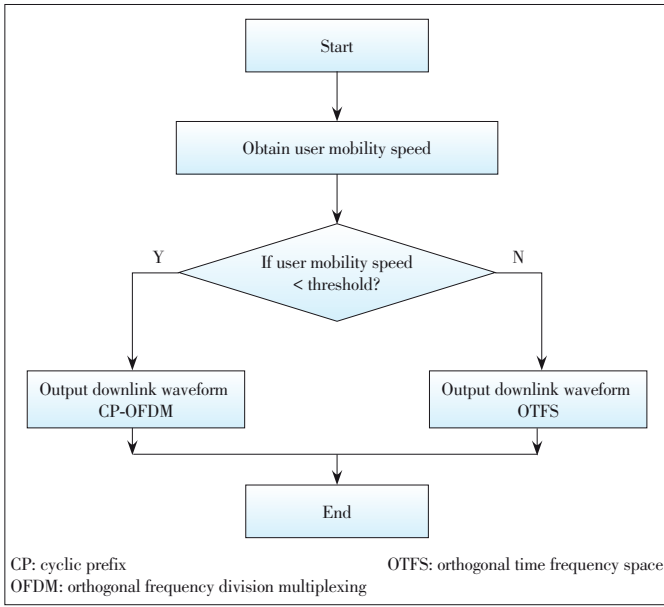
▲ Figure 6. Hybrid OTFS-OFDM multi-waveform detector structure: (a) single user OTFS system; (b) multi-user OTFS system

ing to certain algorithms with the input of user mobility speed and user type. And then such waveform information is carried by the RRCReconfiguration message and is configured to UE through the air interface. As a result, the base station and UE perform the same waveform processing. Comparing single user hybrid OTFS-OFDM systems and multi-user hybrid OTFS-OFDM systems, the main difference and difficulty are in the base station. In multi-user hybrid OTFS-OFDM systems, since the base station supports multi-user transmission and users may adopt different waveforms, the base station should have the capability of processing multiple waveform in parallel.

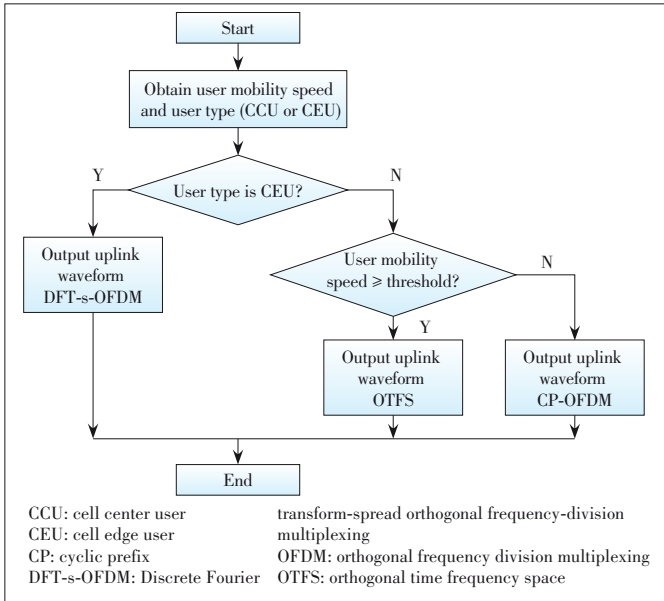
Fig. 7 shows the downlink waveform selection procedure, which will select CP-OFDM or OTFS according to user’s mobility speed. Since OTFS does not show excellent performance in low mobility, the downlink adopts CP-OFDM waveform when user’s mobility speed is lower than a certain speed threshold, otherwise, the downlink adopts OTFS waveform.

Fig. 8 shows the uplink waveform selection procedure, which will select CP-OFDM, DFT-s-OFDM or OTFS according to user’s mobility speed and user type. Since UE has strict requirements for waveform’s PAPR, the uplink adopts DFT-s-OFDM with low PAPR when UE is a cell edge user (CEU). When UE is a cell center user (CCU), the uplink adopts CP-OFDM waveform if user mobility speed is lower than a certain speed threshold, otherwise, the uplink adopts OTFS waveform.

Considering multi-user and multi-waveform communication systems, the base station needs to simultaneously process multiple users with different waveform types, which requires to design multiple access for multi-waveform multi-user systems. Taking downlink transmission for an example, Fig. 9 shows two multiple access schemes for downlink hybrid OFDM-OTFS multi-user systems. In Fig. 9(a), the resource of each user allocated in the TF domain is orthogonal and OTFS users’ resource in the DD domain is also orthogonal, which can effectively avoid inter-user interference. In Fig. 9(b), the T-F resource are firstly divided into two parts, in which one part is for CP-OFDM users and the other part is for OTFS users. Then, CP-OFDM users occupy different T-F resources; while OTFS users are spread in the total T-F resources allocated to all the OTFS users, each OTFS user is orthogonal in the DD domain. Comparing the schemes shown in Figs. 9(a) and 9(b), OTFS users in Fig. 9(a) suffer from less inter-user interference, as they are orthogonal in both DD and T-F domains. However, their diversity gain is also lower than that in Fig. 9



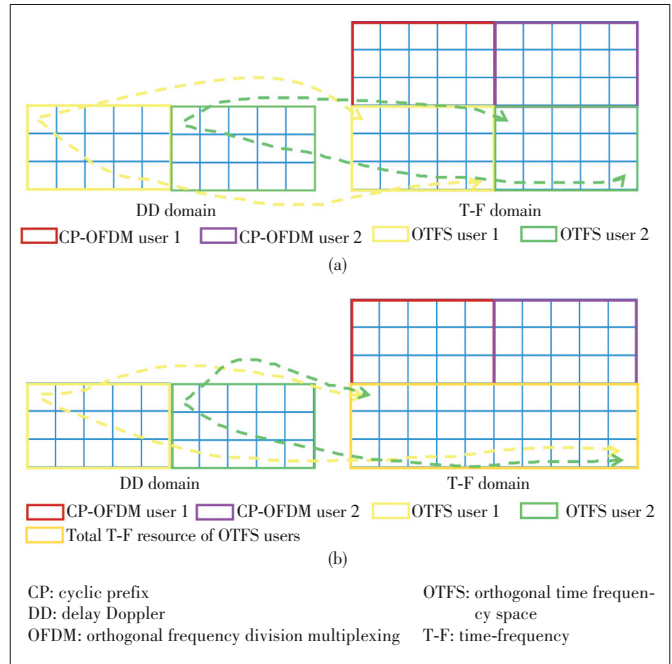
▲ Figure 7. Downlink waveform selection procedure



▲ Figure 8. Uplink waveform selection procedure

(b), as diversity gain of OTFS modulation is positively related with the number of resources. Obviously, the number of resources allocated to each OTFS user in Fig. 9(a) is less than that in Fig. 9(b). For UL multi-waveform multi-user systems, they are similar as the downlink situation.

Fig. 10 shows the block error rate (BLER) performance of two-user OTFS systems without inter-user interference (IUI) and two-user hybrid OTFS-OFDM systems in fractional Doppler channels, where the speed of User 1 is 500 km/h, while the speed of User 2 is 10 km/h. In Fig. 10(a), both User 1 and User 2 adopt OTFS modulation, while in Fig. 10(b), User 1 adopts OTFS modulation but User 2 adopts OFDM modulation. The results show that the OTFS modulation cannot



▲ Figure 9. Multiple access schemes for downlink hybrid OFDM-OTFS systems in multi-user scenario: (a) hybrid orthogonal frequency division multiple access (OFDMA) and orthogonal time frequency space multiple access (OTFSMA) in both DD and TF domains; (b) hybrid OFDMA and OTFSMA with overlap in the TF domain

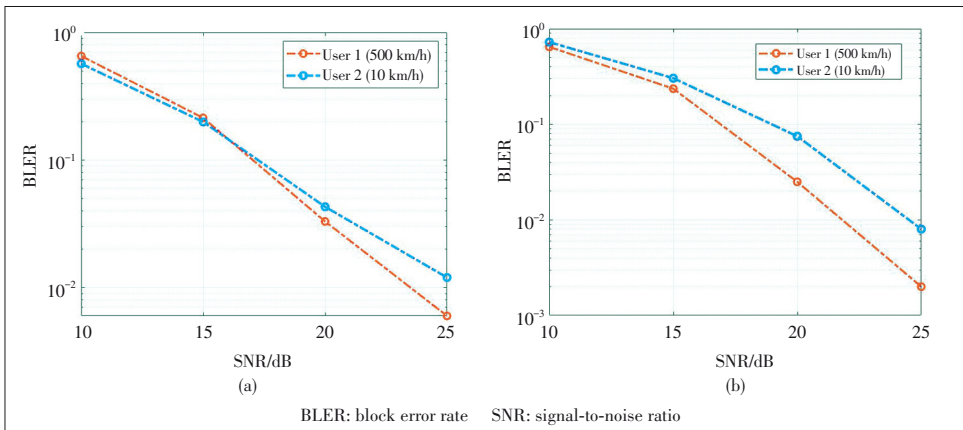
achieve dramatic BLER performance gain in a low-speed scenario while some BLER performance gain can be obtained in a high-speed scenario. Therefore, it is suggested that low-complexity OFDM modulation is used to low-speed users, while high-speed users adopt OTFS modulation. Since different users with different speeds coexist in the base station, hybrid OTFS and OFDM systems should be considered.

## 6 Main Challenges and Future Research Directions

### 6.1 Main Challenges

#### 6.1.1 Low Complexity OTFS Detection Algorithms

In OTFS systems,  $NM$  symbols in the DD domain are spread to the TF domain by employing ISFFT transform, which results in the large number of dimensions of the equivalent DD channel. Furthermore, with the introduction of multi-antenna transmission, the complexity of OTFS detection will also increase dramatically. According to current research results, the minimum computational complexity of OTFS detector is  $O(MN \log(N))$ . Obviously, the computational complexity of OTFS detector is still far higher than that of OFDM detector. As a result, current OTFS detection algorithms cannot satisfy the requirements for OTFS systems. Additionally, many works assume that the DD channel matrix is block circulant and sparse. However, as shown in some works, if there are



▲ Figure 10. BLER performance: (a) two-user orthogonal time frequency space (OTFS) systems without inter-user interference (IUI); (b) two-user hybrid OTFS-orthogonal frequency division multiplexing (OFDM) systems

rich scattering or a large number of paths such as MIMO-OTFS systems, the block circulant and sparsity will not be satisfying. Furthermore, integer Doppler shifts are assumed in many works, while the assumption of fractional Doppler shifts is more reasonable in practical OTFS systems. However, fractional Doppler shifts will increase computational complexity and result in more serious inter-Doppler interference. Therefore, the research on low complexity OTFS detection algorithm is a great challenge.

### 6.1.2 Decoupling Between MIMO-OTFS Detector and Precoder

Current research on OTFS receivers mainly focuses on SISO-OTFS systems, while just a few works study MIMO-OTFS systems. However, when extending SISO-OTFS detection algorithms to MIMO-OTFS systems, it will face some new problems. For example, when the MRC detector is used to MIMO-OTFS systems, it needs to obtain the precoding matrix. However, when a non-codebook-based precoding scheme is adopted, it is difficult for the OTFS receiver to obtain the precoding matrix. That is, in order to match the detection algorithms, some detectors require special design at the MIMO-OTFS transmitter side. This strong coupling design between the MIMO-OTFS receiver and transmitter reduces the flexibility of MIMO-OTFS system design and processing. Therefore, the research on MIMO-OTFS detectors and detection algorithms, which is decoupled with the MIMO-OTFS transmitter, is another challenge.

### 6.1.3 Multi-Waveform Hybrid OTFS Detector

OTFS modulation can obtain delay-Doppler diversity gain, and thus it can achieve better performance than conventional OFDM systems in high mobility scenarios. However, OTFS modulation cannot obtain obvious performance gain in low mobility scenarios. When users experience different scenarios, it would be better to switch waveform to obtain better performance. The coexistence of multiple waveforms, such as OFDM and OTFS, requires that the receiver supports multi-

waveform processing and waveform switching, which increases the complexity of the receiver. Therefore, designing a low complexity unified multi-waveform receiver is also a challenge.

## 6.2 Further Research Directions

### 6.2.1 Advanced Low Complexity OTFS Detectors and Detection Algorithms

The computational complexity of current OTFS detection algorithms has been reduced to  $O(MN \log(N))$ , but it is still

much higher than the acceptable complexity in practical systems. Therefore, in the future, the first important work is to study advanced low complexity OTFS detector structures and detection algorithms. As for OTFS detector structures, even though some single domain and joint multi-domain OTFS detectors have been studied, there are still some novel OTFS detector structures to be studied such as joint channel estimation and detection. As for OTFS detection algorithms, some non-iterative and iterative detection algorithms have been studied, but their computation complexities are still very high, up to  $O(MN \log(N))$ . The properties of DD channel matrix together with some simplified and approximated matrix operations should be further exploited to develop novel OTFS detection algorithms. Furthermore, iterative detection algorithms can achieve better performance, but they are needed to further analyze the convergence by employing some tools such as extrinsic information transfer (EXIT) chart and design efficient iteration stopping schemes to reduce the number of iterations.

### 6.2.2 Learning-Based OTFS Detectors

Several OTFS detection algorithms have been studied, but they reduce the computational complexity by exploiting the block circulant and sparsity, as well as simplified and approximated mathematical methods. It will become more difficult to find more low complexity OTFS detection algorithms. A learning-based method provides a new way for the OTFS detector, which considers OTFS detection processing as a black box and performs OTFS detection by deploying online learning model trained offline. Currently, there are few research works on learning-based OTFS detector, and thus many efforts are needed to study learning models and performance verification. Therefore, the learning-based OTFS detector is a future research direction.

### 6.2.3 Unified Multi-Waveform Detector Design

If OTFS modulation is adopted, the downlink and uplink of future mobile communication systems will be multi-waveform.

However, there are few works to study the receiver design to support coexistence of multiple waveform. To support multi-waveform systems, the receiver should support signal detection of each waveform. A simple way is to deploy multiple signal detection modules and switch among these detection modules according to configured waveform type. However, this way is inefficient and will increase the processing complexity. A better way is to design a unified multi-waveform receiver, which can flexibly and efficiently support signal detection of different waveform. Therefore, unified multi-waveform receiver design is another future research direction.

## 7 Conclusions

In this paper, the research works on low complexity OTFS detectors have been surveyed comprehensively. Firstly, we present the OTFS system model and basic principles. And then, we focus on low complexity OTFS detector structures, and give the categories and discussions of all surveyed OTFS detector structures. According to different classification regulations, two classifications of OTFS detectors are the single-domain OTFS detector and joint multi-domain OTFS detector; the non-iterative OTFS detector and iterative OTFS detector. As for their performance, the joint multi-domain OTFS detector is superior to the single-domain one, while the iterative OTFS detector is better than the non-iterative one. We also provide an overview on the principles of popular OTFS detection algorithms, and discuss them in terms of complexity and performance. Furthermore, considering the coexistence of multiple waveforms such as OTFS and OFDM, we discuss the design for hybrid multi-waveform detectors in single user and multi-user OTFS systems, and waveform switching procedures and algorithms. Finally, we present main challenges for low complexity OTFS detectors and identify some future research directions.

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