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Nonbinary LDPC BICM for Next-Generation High-Speed Optical Transmission Systems

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Abstract

We propose a nonbinary byte-interleaved coded-modulation scheme with inner and outer turbo-like iterative decoder. The net coding gain is 0.6 dB higher than the state-of-the-art binary single parity check (SPC) low-density parity-check (LDPC) based turbo-product counterpart, with adjustable iterations and lower error-floor. We provide the details of Bahl-Cocke-Jelinek-Raviv (BCJR) based inner code decoder and optimum signal constellation design (OSCD) method in this paper. The single-mode fiber (SMF) channel simulation is also discussed.

Keywords

fiber optics and optical communications; modulation; forward error correction (FEC); coded modulation

1 Introduction

The capacity demands due to Internet traffic growth is increasing exponentially. After the 100 Gb/s Ethernet (100GbE) standard has been adopted, the higher data rate age is coming for the next-generation Ethernet standards, possibly for 400 GbE and 1TbE. The most significant issues of the current optical networks include limited bandwidth resources and nonlinearity effects in transmission. For the given physical links and network equipment, an effective solution to the optical signal noise ratio (OSNR) requirement is based on forward error correction (FEC), as the response to the demands of high speed reliable transmission.

The low-density parity-check (LDPC) codes with large coding gains have been considered as the promising solution to the FEC. However, using the LDPC codes only might encounter error floor issues, especially for very high rate LDPC codes. Hard decision (HD) algebraic codes, such as Bose-Chaudhuri-Hocquenghem (BCH) and Reed−Solomon (RS) code, are typically used for clearing error floors. With the stronger computation capacity, the soft decision (SD) decoding algorithm enables more possibilities. Traditionally, the inner code is LDPC code, the outer code can be a shorter simple code such as BCH and RS. With SD decoding enabled, it has been showed that the reversed order decoding can provide a larger coding gain [1]. Considering the linearity of both inner and outer codes, the coding gain can be further increased with iterative decoding as a turbo product code (TPC). Although the single parity check (SPC) code, a special case of BCH code, cannot guarantee to correct any error in HD decoding, it can be a very good candidate in SD TPC decoding, which increases the accuracy of the input probabilities to LDPC decoder. Recently, this idea has been experimentally verified in [2] and a large coding gain is obtained.

To further improve the coding performance, we consider the nonbinary (NB) coding scheme, which has already been proved to have a larger gain especially for higher modulation formats [3]. The nonbinary LDPC code is getting more popular with the advanced hardware and it is proved to be hardware friendly in [5], in which the SD nonbinary LDPC and HD-RS code is implemented in FPGA. As a consequence, instead of binary SPC and LDPC, we consider a larger Galois Field (GF) with four elements, which is a good compromise between performance and complexity [4], and this method can be generalized to any size GF.

Without putting additional redundancy to the data, extra gain can be obtained by using turbo equalizer (TE) decoding scheme [6]. The number of iterations can be adjusted based on the quality demands. To verify the robustness of the proposed nonbinary scheme, a high nonlinear polarization-division-multiplexed (PDM) single-mode fiber (SMF) transmission has been simulated. We also consider the suboptimal and fast algorithm for the decoder, which significantly decreases the complexity with little reduction in performance.

2 Proposed Tandem-Turbo-Product Nonbinary Coding Scheme

The encoder of the proposed nonbinary (NB) tandem-TPC code is shown in **Fig. 1**. Based on two linear codes, NB LDPC and NB BCH/SPC, we encode each row as a codeword of NB LDPC code and each column is a codeword of NB BCH/SPC code. The overall coded block size is N_{inner}-by-N_{outer} with elements from GF(q). The encoded nonbinary block is then passed to the PDM-16QAM modulator with Gray mapping.

The structure of the decoder is provided in **Fig. 2**. The PDM-16QAM symbol likelihood is calculated for the following tandem-turbo-product decoder. At the first iteration, there is no

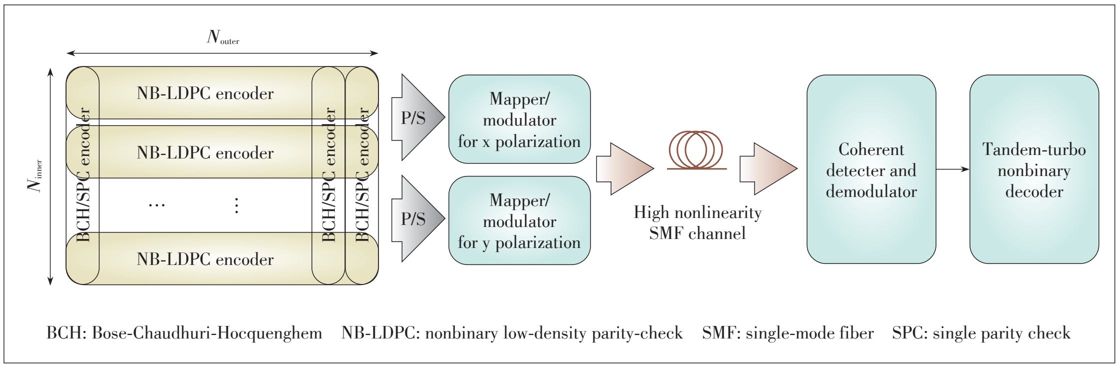


Figure 1. System diagram of the tandem-turbo-product coded modulation scheme over high nonlinearity SMF channel.

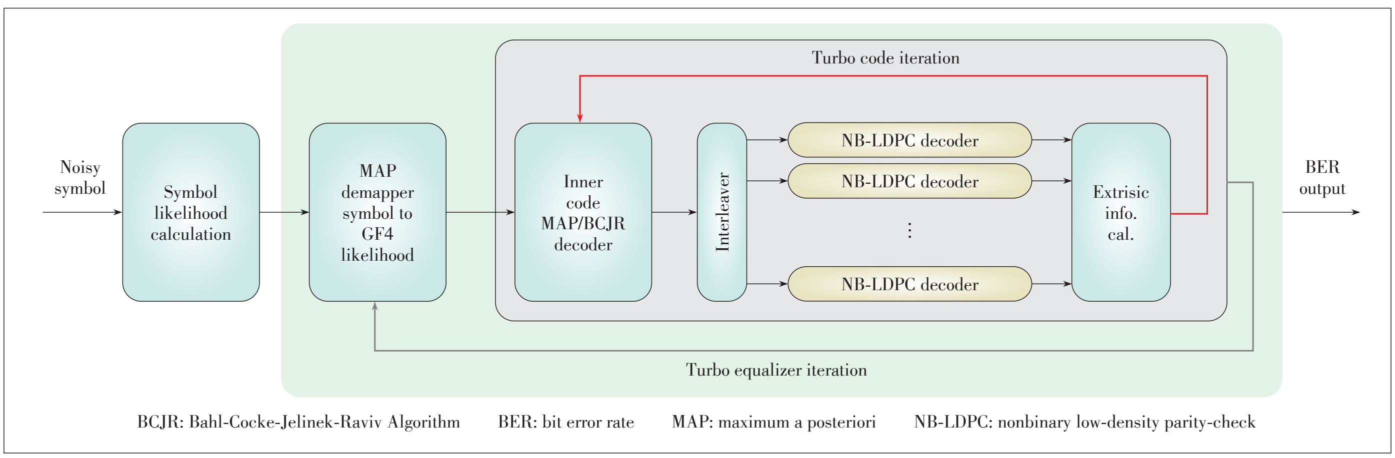


Figure 2. Tandem-turbo-product nonbinary BICM decoder with adjustable iterations.

extrinsic information for the TE, so the demapper simply computes the GF(q) log-likelihood ratio (LLR) for the TPC decoder. The TPC is consisted of two NB linear block codes, BCH and LDPC, and the rows are decoded as LDPC code with row-layered sum-product algorithm and the columns are decoded by maximum a posteriori (MAP) decoder. The MAP decoder can be done with look-up table (LUT) or Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm. The LLRs are updated with both the decoders in each iteration as typical in TPC code. The TE comes to help if the output of the TPC is unsatisfactory at the first round. The extrinsic information of the TPC is remapped to symbol level and attenuated before being combined with the channel symbol likelihood in the demapper. The updated LLRs after the demapper are passed to the TPC decoder and the first TE decoding iteration is started. There are three adjustable iterations, LDPC iteration, TPC iteration, and TE iteration and the trade-off between complexity and performance can be further optimized for further performance improvement.

The symbol log-likelihood ratio (SLLR) of the transmitted symbol can be updated by turbo equalizer as

$$\lambda(s_i)=\log\left(\frac{P(s_i|r)}{P(s_0|r)}\right)=\log\left(\frac{P(r|s_i)P(s_i)}{P(r|s_0)P(s_0)}\right)=\log\left(\frac{P(r|s_i)}{P(r|s_0)}\right)+\lambda_{ext}(s_i), \quad (1)$$

where the s_i is the transmitted symbol ($i=0,1,\ldots,15$ *for* $16QAM$), r is the received symbol and s_0 is the referent symbol point. The second equation is due to Bayes' theorem and $\lambda_{ext}(s_i)$ is the prior reliability of symbol s_i, which can be computed as

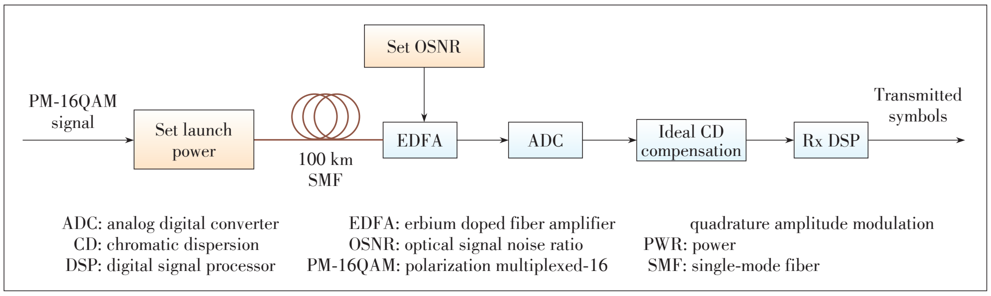
$$\lambda_{ext}(s_i)=\log\frac{P(s_i)}{P(s_0)}=\sum_{j=1,c_j\in s_i}^{m}\log\left(\alpha_t-L_p(c_j)\right), \quad (2)$$

in which $L_p(c_j)$ is the prior GF(q) LLR and c_j is the GF(q) representation of the mapped symbol. α_t is the attenuation coefficient for the t th TE iteration. The prior/extrinsic LLR is the difference between output and input of the TPC decoder as $L_p(c_j)=L(c_j^{out})-L(c_j^{in})$ and the GF(q) LLR for $k\in\{0,1,\ldots,q-1\}$ can be obtained by

$$L(c_j=k)=\log\frac{\sum_{c_j=k,c_j\in s_i}\lambda(s_i)}{\sum_{c_j=0,c_j\in s_0}\lambda(s_0)}. \quad (3)$$

The summation in $L(c_j)$ in (3) can be calculated by max* [7] operation or be simplified by replacing the max* operation by max-operation only, by ignoring the correction terms, which can significantly reduce the complexity. The TPC decoder accepts the GF(4) LLRs and begin the TPC iteration, and in each

TPC iteration, the inner code decoder and LDPC decoder exchange decoded LLRs to improve the performance. The (vertical) inner code is NB SPC/BCH code with BCJR decoder, the trellis decoding algorithm is based on MAP decoding rule, which is of low complexity for SPC. The (horizontal) outer code is NB LDPC code, and it is decoded by row-layered sum-product algorithm.



▲**Figure 3. PM-16QAM SMF transmission link setup with high nonlinearity effect.**

3 SMF Modeling and Channel Simulation

The SMF is modeled by solving the nonlinear Schrödinger equation (NLSE) with split-step (Fourier) method. The total length of SMF is 100 km with step size 100 m, the attenuation is 0.2 dB/km, the dispersion coefficient is 17 ps/nm/km and the nonlinear parameter is $1.2\,(\mathrm{W\cdot km})^{-1}$. Simulation of the channel is done for both Additive White Gaussian Noise (AWGN) channel and SMF-based transmission system (**Fig. 3**). For SMF simulation, the transmitter side uses eight times up-sampling with ideal pre-filter. The power of the PDM-16QAM signal is set before launching into the SMF. A Gaussian noise is loaded after the transmission of SMF with a given OSNR. On the receiver side, the ideal chromatic dispersion (CD) compensation is done before the typical digital signal processor (DSP) blocks and the transmitted symbols are passed to the tandem-turbo-product decoder.

4 Optimal Signal Constellation Design and Performance

New constellation points are obtained as the center of mass of such obtained clusters. This procedure is repeated until convergence or until the predetermined number of iterations has been reached. It can be shown that this algorithm is optimum in minimum mean square error (MMSE) sense.

The MMSE-optimum signal constellation design (OSCD) algorithm can be formulated as follow:

1) Initialization: Choose an arbitrary auxiliary input distribution. Choose an arbitrary signal constellation as initial constellation and set the size of this constellation to M.
2) Apply the Arimoto-Blahut algorithm to determine optimum source distribution.
3) Generate long training sequences $\{x_j; j=0, \cdots, n-1\}$ from optimum source distribution, where n denotes the length of the training sequence used for signal constellation design. Let A_0 be the initial M-level signal constellation set of subsets of constellation points.
4) Group the samples from this sequence into M clusters. The membership to the cluster is decided by Euclidean distance squared of sample point and signal constellation points from previous iteration. Each sample point is assigned to the cluster with smallest distance squared. Given the mth subset (cluster) with N candidate constellation points, denoted as $\hat{A}_m=\{y_i\,;\, i=1, \ldots, N\}$, find the MMSE of partition $P(\hat{A}_m)=\{S_i\,;\, i=1, \ldots, N\}$, as follows

$$D_m = D\left(\left\{\hat{A}_m, P\left(\hat{A}_m\right)\right\}\right) = n^{-1}\sum_{j=0}^{n-1}\min_{y\in\hat{A}_m} d(x_j, y), \tag{4}$$

where d is Euclidean distance squared between the jth training symbol and symbol y being already in the subset (cluster). With $D(\cdot)$, we denote the distance function.

5) If the relative error $|D_{m-1}-D_m|/D_m \leqslant e$, where e is the desired accuracy, the final constellation is described by $\{\hat{A}_m\}$. Otherwise continue.
6) Determine the new constellation points as the center of the mass for each cluster. With the mean square-error criterion, $x(S_i)$ is the Euclidean center of gravity or centroid given by

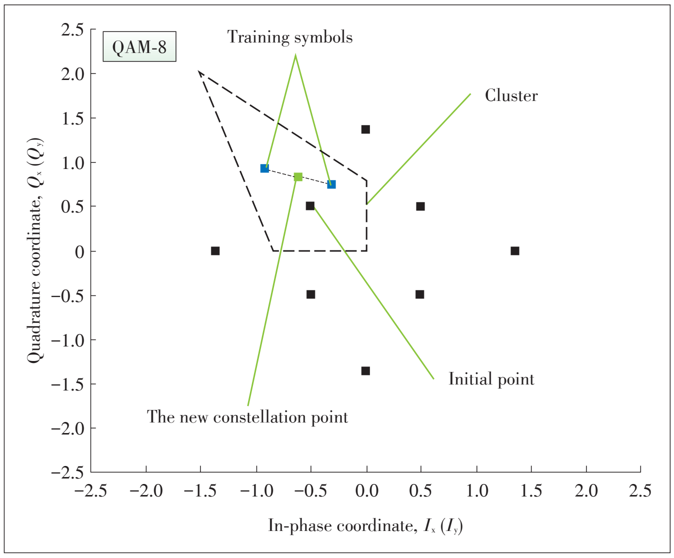
$$x(S_i) = \frac{1}{\|S_i\|}\sum_{j:x_j\in S_i} x_j\,, \tag{5}$$

where $\|S_i\|$ denotes the number of training symbols in the region S_i as shown in **Fig. 4**. If there is no training sequence in the region, set $x(S_i)=y_i$, the old constellation point. Define $\hat{A}_{m+1}=x(P(\hat{A}_m))$, replace m by $m+1$, and go to step 3.

Repeat the steps 4–6 until convergence.

Fig. 5 shows the signal constellations obtained for following signal constellation sizes: 16, 32, 64 and 128. The results are obtained for ASE noise dominated scenario.

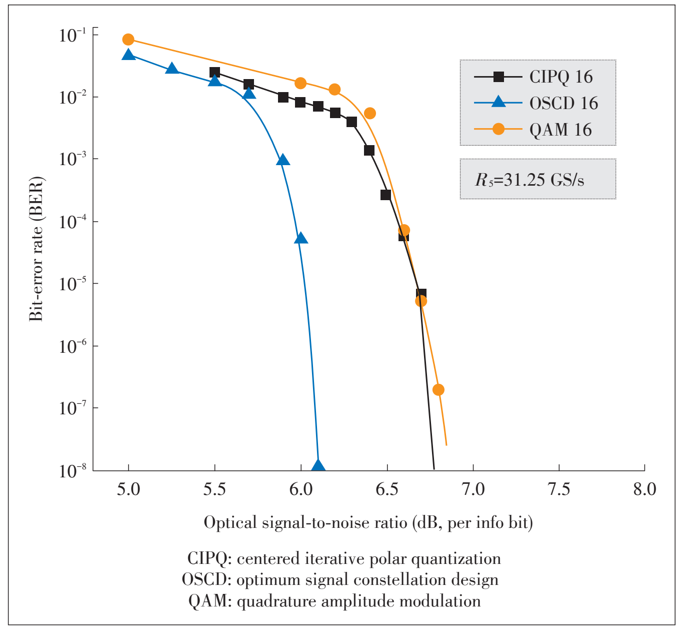
It should be noticed that these signal constellations remind to the format of IPQ-signal constellations, except for the center point. Alternatively, the IPQ-approach can be used by placing the first single point in the origin and then the IPQ-procedure is applied. Howeverthat the IPQ-procedure uses some approximations to come up with closed form solutions, which are valid assumptions for reasonable large signal constellation sizes. Therefore, it is a suboptimum solution for medium signal constellation sizes. We will later show that signal constellations obtained by MMSE-OSCD algorithm significantly outperform IPQ-inspired signal constellations containing the point located in



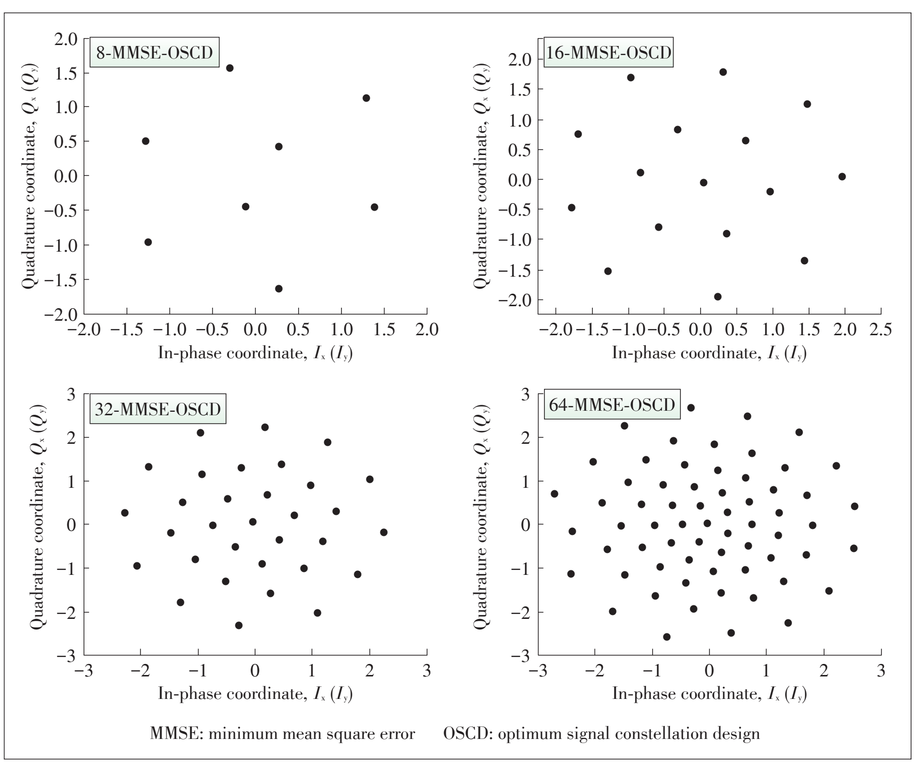
▲Figure 4. The illustration of the OSCD algorithm.

the origin.

The results of Monte Carlo simulations of proposed MMSE-OSCD-algorithm based constellations are summarized in **Fig. 6**. When measured at BER of 10^{-8}, the 16-ary MMSE-OSCD algorithm based signal constellation outperforms 16-QAM by almost 1 dB. The channel symbol rate was set to 31.25 GS/s, and QC LDPC (16935, 13550) of girth-8 and column-wieght-3 was used in simulations. The improvement of MMSE-OSCD over centered iterative polar quantization (CIPQ) and quadrature amplitude modulation (QAM) decreases as the signal constellation size grows.



▲Figure 6. BER performance of proposed MMSE-OSCD algorithm based constellations against QAM and CIPQ.



▲Figure 5. The MMSE-OSCD constellation.

5 MAP/BCJR Decoder for the Inner Code

Optimum or sub-optimum MAP algorithms are usually good candidates to be used in "soft-in/soft-out" decoders. However, the MAP algorithm, especially for nonbinary codes, is a computationally complicated decoding method. Here we present a near optimum MAP decoding rule for nonbinary codes based on the dual space of the code. Because the complexity of this proposed algorithm is related to the inverse of the code rate, it can be attractive for the codes with high coding rates. The shorter code word length is also a plus for implementation consideration.

Here we focus on the inner code only and start with some notation introduction. Vectors and matrices are denoted in boldface letters and their elements in lower case, e.g., v_i is the i th element of the vector $\mathbf{v}=(v_0,v_1,\cdots,v_{N-1})$. Also an ($N$, K) code is referred to a linear block code with the length of N and dimension of K with the parity-check matrix that is denoted by $\mathbf{H}$. The codes are defined over a finite field of order q, F_q, therefore the elements of codewords belong to set {0,1, ... , q-1}. Codewords are transmitted over a discrete time memory-less and noisy channel, and the soft decision received vector for a codeword is denoted by $r=(r_0,r_1,\dots,r_{N-1})$.

The APP algorithm for the block code can be developed as

$$\hat{v}_i=\arg\max_{v_i\in\{0,1,\cdots,q-1\}}\Pr(v_i|r,v\cdot H^T=0) \quad (6)$$

Based on typical BCJR algorithm, the equation above is equal to

$$\hat{v}_i=\arg\max_{v_i\in\{0,1,\cdots,q-1\}}\sum_{(s',s)\in\varepsilon^i_{v_i}}\alpha_i(s')\gamma_i(s',s)\beta_{i+1}(s), \quad (7)$$

where is a state at the i th-level of the code trellis and is a state at the $(i+1)^{th}$-level of the code trellis. Moreover, $\varepsilon^i_{v_i}$ is the set of all edges of the trellis of the code, which connects the states to the states and corresponds to the code symbol v_i. If $r_{i,j}=(r_i,r_{i+1},\dots,r_{j-1})$, for $0\leqslant i\leqslant j\leqslant N$, we then have

$$\alpha_i(s)=\Pr(s_i=s,r_{0,i}), \quad (8)$$

$$\beta_i(s)=\Pr(r_{i,N}|s_i=s), \quad (9)$$

$$\gamma_i(s',s)=\Pr(s_{i+1}=s,r_i|s_i=s'). \quad (10)$$

Let $\Omega^{(c)}_{i-1}(s)$ be as a set of all states at the $(i-1)\,th$-level of the code trellis, which are adjacent to state s. According to the BCJR algorithm for state at ith-level of the trellis, and can be calculated recursively.

For $0\leqslant i\leqslant N$,

$$\alpha_i(s)=\sum_{s'\in\Omega^{(c)}_{i-1}(s)}\alpha_{i-1}(s')\gamma_i(s',s). \quad (11)$$

By knowing the fact that $\alpha_o(s_0)=1$, all α_i for $0\leqslant i\leqslant N$ can be calculated. This is called the forward recursion. The backward recursion is similarly obtained.

For $0\leqslant i\leqslant N$,

$$\beta_i(s)=\sum_{s'\in\Omega^{(d)}_{i+1}(s)}\gamma_i(s,s')\beta_{i+1}(s'), \quad (12)$$

where $\Omega^{(d)}_{i+1}(s)$ is a set of all states at $(i+1)$ th-level of the code trellis and these states are adjacent to state s. Using the fact that $\beta_N(s_f)=1$, β_i all $0\leqslant i\leqslant N$ for can be found. The branch transition probability for a block code with statistically independent information bits can be written as

$$\gamma_i(s,s')=\Pr(s_{i+1}=s,r_i|s_i=s')= \Pr(s_{i+1}=s|s_i=s')\Pr(r_i|s_{i+1}=s,s_i=s')= \Pr(v_i)\Pr(r_i|v_i). \quad (13)$$

Different methods have been suggested to carry out the MAP decoding algorithm, but in all of these methods, there are three common major steps:

1) Perform the forward recursion process and store all the values calculated for α_i for $0\leqslant i\leqslant N$.
2) Perform the backward recursion process and store all the values calculated for β_i for $0\leqslant i\leqslant N$.
3) For each received codeword symbol, r_i, find the MAP probability using (2). To calculate (2), the transition probabilities are needed.

Due to the independence of forward and backward recursions from each other, step 1 and step 2 can be done simultaneously.

6 Results and Discussion

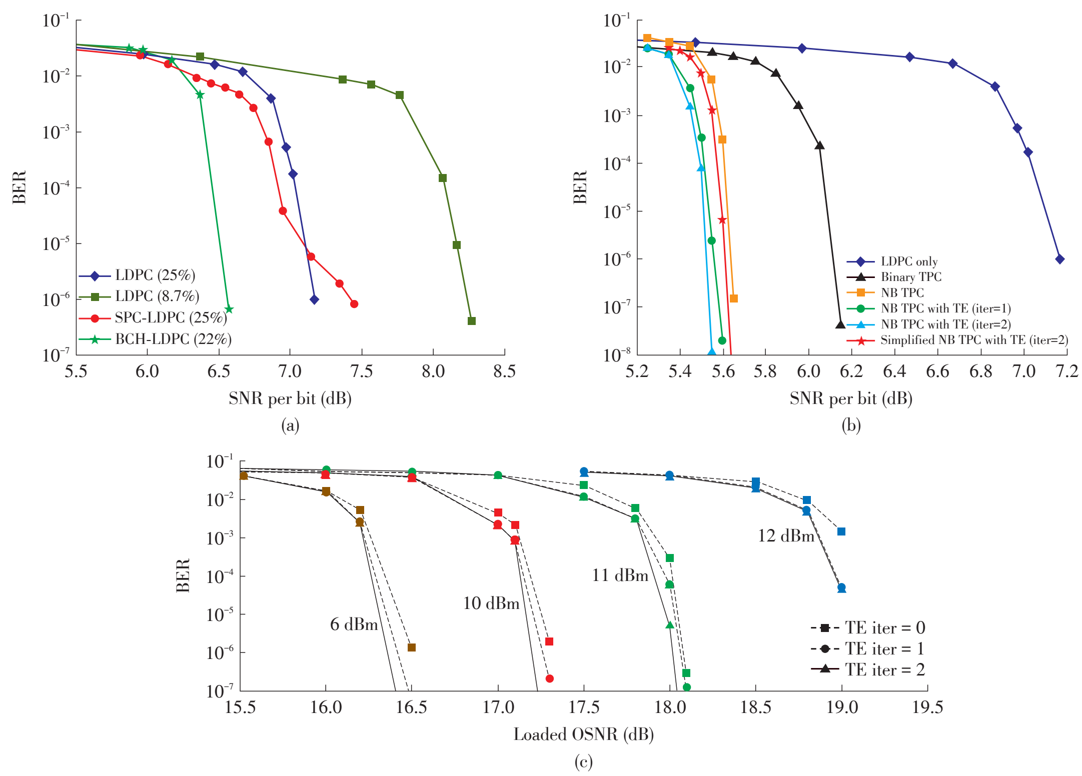
For binary coding, **Fig. 7a** shows that in the BER performance TPC (SPC (7,8) and LDPC (17104,18611)) of 25% overhead outperforms LDPC (17104,18611). Though this TPC performs even better than the LDPC (13550,16935) with the same overhead in the low SNR region, it begins to show some error floor at $BER=10^{-5}$. By replacing the SPC with BCH (57,64), about 0.3 dB gain is obtained compared to the SPC-LDPC case without any error floor and with 22% overhead.

In **Fig. 7b**, the nonbinary TPC (NB-SPC (7, 8) and NB-LDPC (6744, 8430)) has about 0.4 dB gain in OSNR compared to the binary counterpart (SPC (7,8) and LDPC (13550, 16935)) with half the length. With two TE iterations, we are able to extend the gain to 0.6 dB without increasing the overhead. It should be noticed that both binary and nonbinary performance are evaluated at the 4-th TPC iteration, and the maximum number of iterations for LDPC of are 30 and 18 for binary and nonbinary LDPC codes, respectively. When reducing the max*-operation to max-operation of the nonbinary TPC code with TE, only less than 0.2 dB loss in SNR is observed.

With four different launch powers, ranging from 6 dBm to 12 dBm, the proposed tandem-TPC scheme is tested with studied SMF link. After the transmission, to achieve the same BER performance, the higher linear ONSR is required for larger nonlinear transmission cases. **Fig. 7c** shows the BER performance of nonbinary twin-turbo code (NB-SPC (7, 8) and NB-LDPC (6744, 8430)) with TE, and it demonstrates excellent robustness even in highly nonlinear scenarios.

7 Conclusions

We proposed the nonbinary tandem-TPC-TE based coded modulation scheme. When TE and TPC are employed in tandem, a smaller number of iterations are needed for NB LDPC with half the length compared to the binary counterpart. The proposed NB TPC provides 0.6 dB improvement in Net Coding Gain (NCG) for the same BER performance with adjustable iteration/performance. The complexity is even lower when low-complexity decoding algorithm is used with small loss in gain.



BCH: Bose-Chaudhuri-Hocquenghem
BER: Bit Error Rate
LDPC: Low-Density Parity-Check
NB: Non-Binary
OSNR: Optical Signal Noise Ratio
SMF: single-mode fiber
SNR: Signal Noise Ratio
SPC: single parity check
TE: turbo equalizer
TPC: turbo product code

Figure 7.
BER performance in:
(a) BCH vs SPC based TPC;
(b) nonbinary TPC vs binary TPC; and
(c) SMF nonlinear transmission with loaded linear noise.

The robustness has been verified in highly nonlinear PDM-16QAM SMF transmission. The proposed NB TPC represents a promising NB coded-modulation scheme for the next generation optical transmission systems.

References

[1] I. B. Djordjevic, L. Xu, and T. Wang, "Reverse concatenated coded modulation for high-speed optical communication," *IEEE Photonics Journal*, vol. 2, no. 6, pp. 1034–1039, Dec. 2010. doi: 10.1109/JPHOT.2010.2091678.

[2] H. G. Batshon and H. Zhang, "Multidimensional SPC-based bit-interleaved coded-modulation for spectrally-efficient optical transmission systems," in *Proc. SPIE 9008, Optical Metro Networks and Short-Haul Systems VI, 90080F*, San Francisco, USA, 2013. doi: 10.1117/12.2037021.

[3] M. Arabaci, I. B. Djordjevic, L. Xu, and T. Wang, "Nonbinary LDPC-coded modulation for high-speed optical fiber communication without bandwidth expansion," *IEEE Photonics Journal*, vol. 4, no. 3, pp. 728–734, Jun. 2012. doi: 10.1109/JPHOT.2012.2195777.

[4] D. Zou and I. B. Djordjevic, "FPGA implementation of concatenated non-binary QC-LDPC codes for high-speed optical transport," *Optics Express*, vol. 23, no. 11, pp. 14501–14509, May 2015. doi: 10.1364/OE.23.014501.

[5] Y. Zhang and I. B. Djordjevic, "Multilevel nonbinary LDPC-coded modulation for high-speed optical transmissions," in *Asia Communications and Photonics Conference*, Shanghai, China, Nov. 2014. doi: 10.1364/ACPC.2014.ATh1E.6.

[6] M. Cvijetic and I. B. Djordjevic, *Advanced Optical Communication Systems and Networks*. Boston, USA: Artech House, Jan. 2013.

[7] C. Lin, I. B. Djordjevic, and D. Zou, "Achievable information rates calculation for optical OFDM few-mode fiber long-haul transmission systems," *Optics Express*, vol. 23, no. 13, pp. 16846–16856, 2015. doi: 10.1364/OE.23.016846.

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