DOI: 10.3969/j. issn. 1673-5188. 2016. S1. 007
http://www.cnki.net/kcms/detail/34.1294.TN.20161229.1035.002.html, published online December 29, 2016

One Step Hologram Calculation for Holographic 3D Display Based on Nonuniform Sampled Angular Spectrum Method

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Abstract

We proposed a method for calculating the computer generated hologram from multi-plane 3D objects by using nonuniform sampled angular spectrum method (NUASM). Both of the hologram plane and the image plane are nonuniform sampled according to the distances and positions of the three-dimensional objects. The nonuniform fast Fourier transform (NUFFT) is used to calculate the angular spectrum propagation from the image plane to the hologram plane and the hologram can be calculated in only one step. Simulation and optical experiment results show that the hologram generated in this way can reconstruct objects on multi-planes simultaneously and separately without axial distortion.

Keywords

holographic display; computer holography; nonuniform fast Fourier transform

This work is supported by Major State Basic Research Development Program of China under Grant No. 2013CB328803, National High Technology Research and Development Program of China under Grant No. 2012AA03A302 and No. 2013AA013904, and ZTE Industry-Academia-Research Cooperation Funds.

1 Introduction

In the field of holographic three dimensional displays, the calculation of computer generated hologram is a big problem for its large amount of points and long calculation time. In order to speed up the calculation, many methods have been proposed, such as the look-up table method [1], [2], the polygon-based method [3], [4] and the wavefront recording plate method [5], [6]. These methods are very useful with those complicated objects generated by computer graphics software, but not suitable for objects with large depth. For some simple situations of hologram calculation, the objects can be divided into several planes and the hologram can be calculated by the diffraction from multi-planes [7], in which the fast Fourier transform (FFT) algorithm can be used to accelerate the calculation between parallel planes.

In the calculation of hologram for multi-plane objects, the diffraction between parallel planes can usually be calculated by either the Fresnel diffraction or the angular spectrum method [8]–[12]. For the Fresnel diffraction, there are two main problems. First, in order to obtain the light distribution in the hologram plane, the diffraction should be calculated from each plane to the hologram separately by many times. This even spends more time when it combined with the iteration algorithm (such as GS iteration) for the kinoform generation. Second, due to the restriction of the sampling rate in the calculation by sampling theory, there is axial distortion in the reconstruction, which will cause the deformation of the objects. For the angular spectrum method, there is no axial distortion in the reconstruction because the sampling rates in both planes are the same in the calculation. However, it still needs many times of diffraction calculation from each plane to the hologram. In this paper, we proposed a method to calculate the hologram from multi-plane objects based on the nonuniform sampled angular spectrum method and the diffraction calculation from objects to hologram is calculated in only one step instead of many steps in the conventional method.

2 Nonuniform Sampled Angular Spectrum Method

2.1 Basic Principle for Changing the Sampling Rate in Angular Spectrum

The angular spectrum method is usually used to calculate the diffraction between parallel planes with the same sampling rate in near field. For example, the calculation from hologram plane to image plane using angular spectrum is expressed as

$$f(x)=FFT^{-1}\left[FFT\big(h(u)\big)\cdot \exp\left(i2\pi z\sqrt{\frac{1}{\lambda^2}-f^2}\right)\right], \quad (1)$$

where FFT and FFT^{-1} represent the Fourier transform and inverse Fourier transform, $h(u)$ and $f(x)$ are the distribution on hologram and image planes. z is the distance between two planes and l is the wavelength. f is the spectrum coordinate in the Fourier domain. The calculation of angular spectrum is composited of two FFTs and one exponential multiplication. Now let us analyze the exp term in (1). By using the Binomial approximation, the exp term can be approximated as

$$\exp\left(i2\pi z\sqrt{\frac{1}{\lambda^2}-f^2}\right)\approx\exp\left(\frac{i2\pi z}{\lambda}\left(1-\lambda^2 f^2\right)\right)=$$
$$\exp\left(\frac{i2\pi z}{\lambda}\right)\cdot\exp\left(-i2\pi z\lambda^2 f^2\right). \tag{2}$$

In the right of (2), the first term has nothing to do with the inverse Fourier integral, so it can be discarded and we only consider the second term $\exp(-i2pzl^2f^2)$. Let the sampling rate of the hologram plane be Δu, so according to the sampling theory of the Fourier transform, the sampling rate in the Fourier domain is $\Delta f=1/(N\Delta u)$, where N is the total number of the sampling points. So (1) can be rewritten in the discrete form as

$$f(n\Delta x)=FFT^{-1}\left[FFT\left(h(m\Delta u)\right)\cdot\exp\left(\frac{i2\pi z\lambda^2}{N^2\Delta u^2}\right)\right] \tag{3}$$

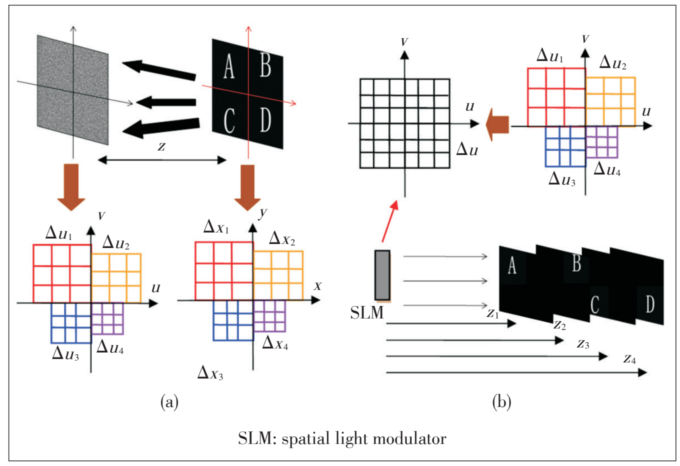
where m and n represent the number of sampling points. Now if the sampling rate in hologram plane is changed from Δu to $\Delta u'=a\Delta u$, (3) can be rewritten as

$$f(n\Delta x)=FFT^{-1}\left[FFT\left(h(m\Delta u)\right)\cdot\exp\left(\frac{i2\pi\alpha^2 z\lambda^2}{N^2\alpha^2\Delta u^2}\right)\right]=$$
$$FFT^{-1}\left[FFT\left(h(m\Delta u)\right)\cdot\exp\left(\frac{i2\pi z'\lambda^2}{N^2\Delta u'^2}\right)\right] \tag{4}$$

Eq. (4) means if the sampling rate of hologram is scaled by a, the image will be reconstructed at a different distance $z'=a^2z$.

2.2 Method of Hologram Design for Multi-Plane Objects

Let us assume a three dimensional object is composed of four letters and each letter is located in different distance from the hologram. The two dimensional pictures with $N\times N$ resolution of this object is taken and located in the image plane. The distance between the image and hologram is z. Both of the hologram plane and image plane (**Fig. 1a**) are decomposed into four parts according to the four quadrants of the coordinate system with the same resolution $N/2\times N/2$. But the sampling in each quadrant is not the same. From the first quadrant to the fourth quadrant, the sampling rate in sequence is $u_2=a_2\Delta u$, $u_1=a_1\Delta u$, $u_3=a_3\Delta u$ and $u_4=a_4\Delta u$, where Δu is the sampling rate of the spatial light modulator (SLM). Similarly the image plane is sampled in the same way as the hologram plane with four different sampling rate Δx_1, Δx_2, Δx_3 and Δx_4 in each quadrant. In this way, the hologram plane and image plane are nonuniformly sampled and the light distribution in the hologram plane can be calculated by diffraction theory from the image.



▲**Figure 1. (a) Sampling method of the hologram plane and the image plane; (b) Reconstruction process from the nonuniform sampled hologram.**

Fig. 1b shows the reconstruction process from the nonuniform sampled hologram. The hologram is loaded into the SLM which has the same sampling rate in each pixel of u. Therefore, this hologram loading would make a mandatory change in the sampling rate of the hologram from the nonuniform sampling to the uniform sampling, which will cause the change of the sampling rate in each quadrant. The sampling rate change in each part will lead to the reconstruction distance change of each letter. For example, the change of sampling rate from Δu_1 to Δu will give rise to the movement of reconstruction position of letter "A" from z to z_1 (Fig. 1b). So are the same effects for the reconstruction of letter "B", "C" and "D". Therefore, the four letters can be reconstructed at different planes simultaneously and separately by nonuniformly sampling the hologram.

2.3 One Step Calculation of Hologram Using Nonuniform Sampled Angular Spectrum Method (NUASM)

We use the nonuniform fast Fourier transform (NUFFT) for the nonuniform sampled angular spectrum method. The principle of calculation is shown in **Fig. 2**. Fig. 2a is the conventional algorithm for angular spectrum method between uniform sampled planes. It includes one FFT and one inverse FFT to calculate the diffraction from uniform sampled image plane $f(x)$ to hologram plane $h(u)$. But if the sampling of $f(x)$ and $h(u)$ are nonuniform (Fig. 2b), the NUFFT is used to calculate the Fourier transform.

NUFFT is an algorithm for calculating the Fourier transform when sampling is nonuniform in either the spatial domain or the frequency domain [13]–[15]. It has been used in scalar diffraction calculation of nonuniform sampled planes by T. Shimobaba et al. [16]–[18]. Generally NUFFT can be categorized into two types: the calculation from nonuniform sampled points to uniform sampled points, and the calculation from uniform sampled points to nonuniform sampled points. In our situation, the calculation of Fourier transform from image plane $f(x)$ to its

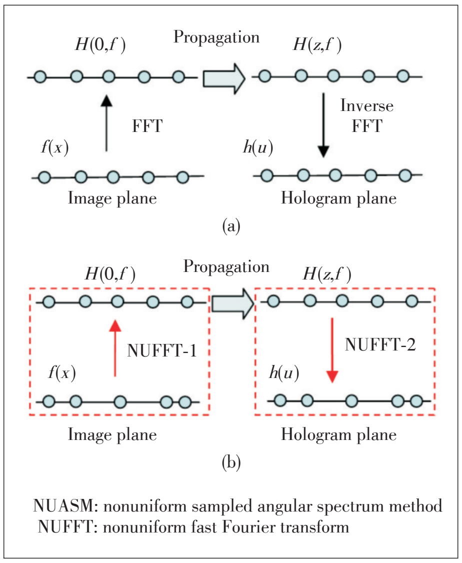


Figure 2. The comparison between (a) conventional angular spectrum method and (b) NUASM.

spectrum domain $H(0,f)$ is the first type. When the spectrum is uniform sampled and it is propagated to the spectrum of distance z to $H(z,f)=H(0,f)\cdot\exp\left(i2\pi z\sqrt{\left(1/\lambda^2\right)-f^2}\right)$, the calculation from $H(z,f)$ to the nonuniform sampled hologram plane $h(u)$ is the second type of the NUFFT . Therefore, the whole calculation of the nonuniform sampled angular spectrum method from the image plane to the hologram plane is expressed as

$$h(u)=NUASM\left[f(x)\right]=$$
$$NUFFT2\left[NUFFT1\left[f(x)\right]\cdot\exp\left(i2\pi z\sqrt{\left(1/\lambda^2\right)-f^2}\right)\right], \quad (5)$$

where $NUASM$ is the symbol for nonuniform sampled angular spectrum method, $NUFFT1$ and $NUFFT2$ represent the type 1 and type 2 nonuniform fast Fourier transform respectively. By using 5 the nonuniform sampled hologram which can reconstruct objects on multi-plane simultaneously can be calculated from the two-dimensional image in only one step, rather than the conventional method in which it needs several steps of calculation proportional to the numbers of object planes.

3 Experiments and Results

3.1 Computer Simulation

We first carried out the computer simulation to demonstrate our method of one step hologram calculation. The three-dimensional object is composed of four Chinese characters with different depths so the four characters are located at different distance from the hologram as shown in **Fig. 3**. At first the four characters are combined into one two-dimensional image of 1024 ×1024 resolution and the image is located at the image plane which has the distance z=100 mm from the hologram plane. Both of the image plane and the hologram plane are divided into four parts according to the four quadrants of coordinate system and are nonuniform sampled in the way introduced in Fig. 1a, and the sampling rates are $u_1=1.3\Delta u$, $u_2=1.1\Delta u$, $u_3=0.9\Delta u$ and $u_4=0.7\Delta u$ (Δu is the sampling rate of the SLM). Then the hologram is calculated by (5) from the image plane to the hologram plane.

Then the reconstruction from the complex hologram is calculated by using the FFT based angular spectrum method, and the sampling rate of the hologram is Δu in the calculation. The four characters cannot be reconstructed at the distance z because of the change of sampling rate. Instead, due to the principle described in section 2.1, the change of the sampling rate will result in the reconstruction of four characters in different planes. The relationship between the new distance and the distance z can be calculated. For the first character, the sampling rate changes from $1.3\Delta u$ to Δu, so the reconstruction distance is changed from z to $(1/1.3)^2 z$. Followed by this, the new reconstruction distance of each character can be calculated by $z_1=z/(1.3)^2$=59 mm, $z_2=z/(1.1)^2$=83 mm, $z_3=z/(0.9)^2$= 123 mm, and $z_4=z/(0.7)^2$=204 mm respectively. The numerical reconstruction result is shown in **Fig. 4a**. It is clear that in each plane the responding Chinese character is clearly focused while the other characters are blurred. Because the sampling rate remains the same in the numerical calculation of FFT based angular spectrum method in reconstruction, the size of the four reconstructed images are the same as the hologram (8 mm×1024). As a comparison, we also presented the simulation results from the compute-generated hologram (CGH) calculated by using the conventional method (**Fig. 4b**). The CGH is calculated and reconstructed by Fresnel diffraction and the size of each reconstructed character is proportional to the distance. It can also be seen from Fig. 4a that all of the four characters are in the same size in each plane and there is no axial distortion with the reconstructed distance which exists in the hologram of multi-plane objects calculated by the Fresnel diffraction. This is because the nonuniform sampling in the image plane makes compensation for the character size in the reconstruction.

3.2 Optical Reconstruction

The optical reconstruction is carried out to show the valida-

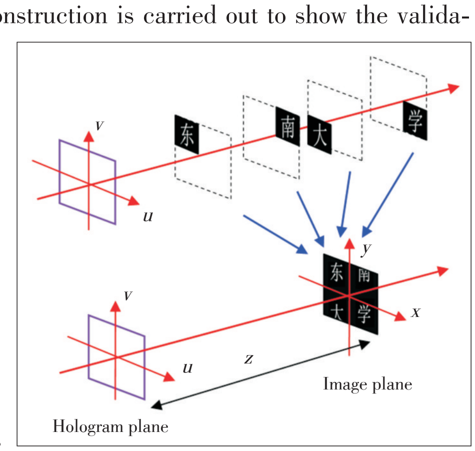
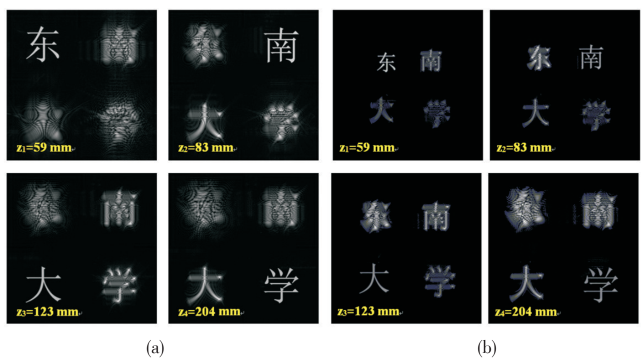


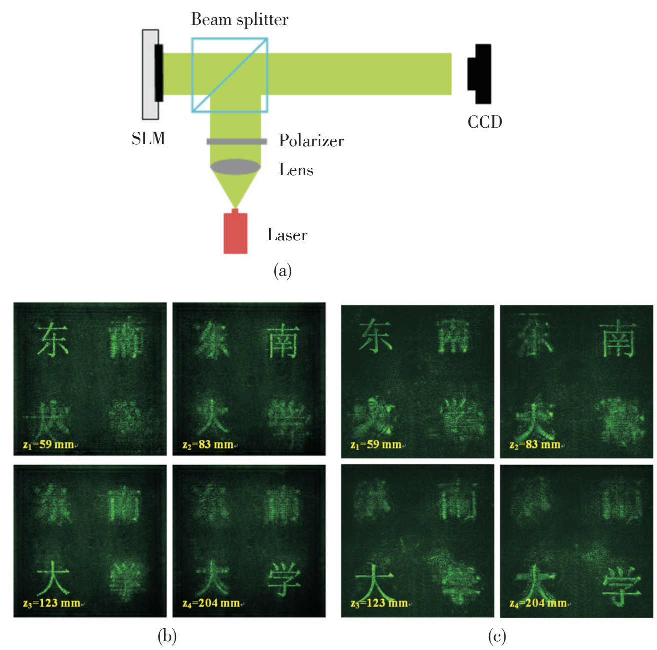
Figure 3. Schematic of one step hologram calculation from 3D object.



▲Figure 4. Reconstruction from nonuniform sampled hologram at different distances by computer simulation: (a) the proposed method and (b) the conventional method.

tion of our method in application. Because we use a phase-only type SLM, it is necessary to encode the complex hologram into the phase-only hologram (kinoform). The phase-only hologram can usually be generated by simply discarding the amplitude term of the complex light distribution in the hologram plane, but this will result in much speckle noise in the reconstructed images due to the direct lose of the amplitude information. Here we use the Gerchberg and Saxton (GS) iteration algorithm for optimizing the hologram [19]. The iteration starts by multiplying an initial random phase to the two-dimensional image. The diffraction from nonuniform sampled image to nonuniform sampled hologram is calculated by the nonuniform sampled angular spectrum method in (5), and then we extract only the phase component and impose unity amplitude constraint. The inverse diffraction starting from hologram is calculated by the nonuniform sampled angular spectrum method again back to the image plane, and the amplitude of the image is imposed while remaining the phase component. By repeating this iterative loop with several times, the hologram can be optimized to be used for the phase-only SLM.

Fig. 5a shows the optical setup for the hologram reconstruction on multi-planes. The 532 nm laser is first collimated by the lens and polarizer and then illuminates the SLM after going through the beam splitter. The SLM is holoeye Pluto with 1920×1080 pixel resolutions and 8 mm sampling rate. The reconstructed images are captured directly by the charge coupled device (CCD) camera at different distances. **Fig. 5b** shows the reconstructed images of four Chinese characters at z_1=59 mm, z_2=83 mm, z_3=123 mm and z_4=204 mm respectively. Each character is clearly focused on four separate planes to achieve the multi-plane reconstruction of the three-dimensional object simultaneously. It can also be seen from the results that the four reconstructed characters are the same in size, so there is no axial scale distortion occurred in the reconstruction, which proves the feasibility of our method for true three-dimensional holographic display. **Fig. 5c** shows the reconstructions from the CGH calculated by using conventional method,



CCD: charge coupled device SLM: spatial light modulator

▲Figure 5. (a) Optical setup for holographic display from nonuniform sampled hologram; (b) reconstructed images of four Chinese characters at different distances by our proposed method; (c) reconstructed images of four Chinese characters at different distances by the conventional method.

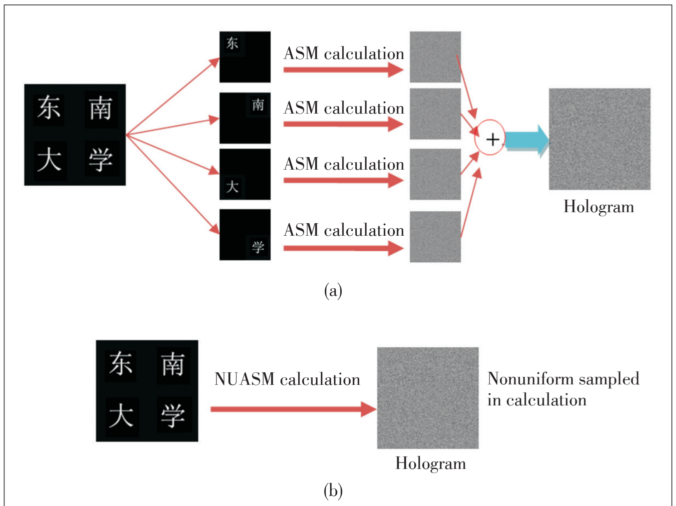
and it can be seen that there is a size distortion of each character due to different reconstructed distances.

4 Discussion

Fig. 6 shows the comparison between the conventional method and our proposed method of hologram calculation for multi-plane 3D objects. In Fig. 6a, the conventional method starts from the decomposition of the three-dimensional objects into several sub-images and then each sub-image is propagated to the hologram plane by the calculation of angular spectrum method. Finally the hologram of multi-plane 3D object is obtained by adding all of the holograms from sub-images. So if the numbers of the object planes are N, it is necessary to calculate the angular spectrum diffraction for N times. In Fig. 6b, by nonuniformly sampling the hologram, we can directly calculate the diffraction from the three-dimensional object to the hologram based on the NUFFT algorithm, and the hologram has the same function for reconstructing at different planes. It is also known that the nonuniform sampled hologram has the advantage of optimizing the numbers of sampling points and eliminating the redundant information properly [20].

5 Conclusions

In conclusion, we proposed a method to calculate the com-



ASM: angular spectrum method NUASM: nonuniform sampled angular spectrum method

▲Figure 6. Comparison of the hologram calculation of multi-plane 3D object: (a) hologram calculated by the conventional ASM and (b) hologram calculated by NUASM.

puter generated hologram of objects on multiple planes by only one step. Both of the hologram plane and the image plane are nonuniformly sampled and the nonuniform sampled angular spectrum method is proposed for diffraction calculation from the image to the hologram. The advantage of our method is that the diffraction calculation from the multi-plane 3D object to the hologram can be calculated in only one step, rather than many steps from each plane to the hologram in the conventional method. Both of the numerical simulation and optical experiment confirmed the practicability of our method. This work has potential applications in the implementation of displaying three dimensional object which can be treated or decomposed into several closely planes.

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Manuscript received: 2015-10-21

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